NEW ITERATION PROCESS FORINFINITE FAMILIES OFEXPANSIONMAPPINGS

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Abstract - In this paper we introduces two new iteration process for different types of mappings and introduce a concept of expansion mapping which is independent of nonexpansive mapping. Also, we study the convergences(weakly and strongly)for these iterations toasymptotic common fixed point and metric projection point respectively in the Hilbert space.

Keywords - Asymptotic fixed point, strong convergence, nonexpansive mapping, maximal monotone , weak convergence.

I. INTRODUCTION

Let be a real Hilbert space and A be multivalued mapping with domain
$D(A) = \{x \in X; Ax \neq \emptyset\}$and $R(A) = \{y \in D; \exists x \in D(A)$such that $y \in Ax\}$.The mapping A is called monotone mapping if:
$< x_1 - x_2 , y_1 - y_1 > \geq 0, \forall x_i \in D(A),$
$\forall y_i \in R(A)$.
Also, any mapping A is called maximal monotone mapping of A if the graph (A) of A is not properly contained in the graph of any other monotone mapping
The monotone mappings play a crucial role in modern nonlinear analysis and optimization, see the books $([1]-[5])$.The convergence of the iteration processes are studied by many researchers see$([6]-[16])$. Consider a single valued non-expansive mapping as follows:
$J_{r_n} = (I + r_n A^{-1})(x)$,which is called resolvet mapping where $< r_n >$ be a sequence of positive real numbers. Also, the metric projection$P_c(x)$ from onto C is defined as follows :
For any $x \in$ there exists a unique element $P_c(x) \in C$ and satisfy the following:
$\|x - P_c(x)\| \leq \|x - y\|$for all $y \in C$.
That is, for each $x \in$, $P_c(x) = y$ iff $y \in C$ and$\|x - y\| = \inf\{\|x - z\|; z \in C\}$.

Now, we recall the following definitions and lemmas

Lemma(1.1) [16]
Let $< \alpha_n >$ and $< \beta_n >$are sequences ofnonnegativenumber such that: $\alpha_{n+1} \leq \alpha_n + \beta_n$, for each$n \geq 1$. If$\sum_{n=o}^{\infty} \alpha_n$converge then
$\lim_{n\to\infty} \alpha_n$exists.
Definition(1.2) : [17]
Let $: C \to C$, then every $p \in C$ is called asymptotic fixed pointof T if there exists $\langle x_n \rangle$ is sequence in C such that $x_n \rightharpoonup p$ and $\|x_n - T(x_n)\| \to 0$.

Lemma (1.3) : [18]
Let C be a nonempty convex closed subset of real Hilbert space Xand T is non-expansive multivalued mapping such that $Fix(T) \neq \emptyset$.Then T is demi-closed, i.e.,$x_n \rightharpoonup p$ and $\lim_{n\to\infty} d(x_n, T(x_n)) = 0$.Then $p \in T(p)$.
Lemma(1.4) : [19]
Let be a Hilbert space and C be a nonempty convex closed subset of if $\langle x_n \rangle$ be a sequence in and $\|x_{n+1} - x\| \leq \|x_n - x\|$ for all $n \in N, x \in C$. Then $\langle P_c(x_n) \rangle$ converges strongly to a point in C.

Now, we introduce the concept of$k-$generalized mapping

II. MAIN RESULTS

Now, we define aniteration processes for sequence of expansion mapping and then study the convergence for these processes.

Definition(2.1)
A mapping f is calledexpansion mapping if for each sequence $\langle z_n \rangle$ in $(0,1)$ converges to 0 there exists an$on-$negative real numberz such that
$(1 - z_n)\|x - w\|^2 + z\langle x - f_x, w - f_w\rangle^{k+1}$
$\geq \|fx - fw\|^2$,for all $k, m > 0$ *and* $x, w \in C$

The concept of expansion mapping is independent of concepts nonexpansive mapping.As shown by the examples:

Example (2.2)
Let be a normed space $f: (0,\infty) \to (0,\infty)$such that $f(x) = 2x$..
the mapping f is not nonexpansive and it is expansion mapping . Then for any sequence $\langle z_n \rangle$in [0,1] converges to 0 there exists z such that,
$z = \frac{4}{\langle x - f_x, w - f_w\rangle^{k+1}}\|x - w\|^2$and satisfy
$(1 - z_n)\|x - w\|^2 + z\langle x - f_x, w - f_w\rangle^{k+1}$
Example (2.3)

Let be a normed space, $f:\ \rightarrow$ suchthat $f(x) = x$.

It is clear that the mapping f is not expansion mappingbut it is nonexpansive

Theorem (2.4):

Let $A_1, A_2, \ldots\ldots, A_m$ are maximal monotone multivalued mapping, C nonempty convex closed in , $\langle f_n \rangle$ be a sequence of non-expansive mapping and $\langle T_n \rangle$ is bounded sequence of expansion mapping on C. Let $\langle a_n \rangle, \langle b_n \rangle$ are sequences in $(0,1]$ converges to 0, such that $a_n + b_n = 1$ and $\sum_{i=1}^{m} \gamma_{n,i} = 1$.Define the iteration process $\langle x_n \rangle$ as follows:

$$y_n = b_n x_n + (1 - b_n) \sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n$$

$$x_{n+1} = a_n[b_n T_n x_n + (1 - b_n) f_n y_n] + (1 - a_n) f_n x_n$$

If $\bigcap_{n=1}^{\infty} Fix\left(J^i_{r_{n,i}}\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right) \neq \emptyset$.Then $\langle x_n \rangle$ has converges weakly to an asymptotic common fixed point of T_n, for each $n \in N$. Moreover $\langle P_C(x_n) \rangle$ converges strongly to the point in C.

Proof :

Let $p \in \bigcap_{n=1}^{\infty} Fix\left(J^i_{r_{n,i}}\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right)$

$$\|y_n - p\|^2 = \left\| b_n(x_n - p) + (1 - b_n) \left(\sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} y_n\ (x_n) - p \right) \right\|^2$$

$$\leq b_n \|x_n - p\|^2 + (1 - b_n) \sum_{i=1}^{m} \gamma_{n,i}\ \|x_n - p\|^2$$

$$\leq b_n \|x_n - p\| + (1 - b_n) \|x_n - p\|^2$$

$$= \|x_n - p\|^2$$

Now, for each sequence $\langle z_n \rangle$in $[0,1]$ converges to 0 there exists a non – negative real number z such that such that

$$\|x_{n+1} - p\|^2 = \|a_n[b_n T_n x_n + (1 - b_n) f_n y_n] + (1 - a_n) f_n x_n - p\|^2$$

$$\leq a_n \|b_n T_n x_n - p + (1 - b_n) f_n y_n - p\|^2 + (1 - a_n) \|f_n x_n - p\|^2$$

$$\|x_{n+1} - p\|^2 \leq a_n\{b_n \|T_n x_n - p\|^2 + (1 - b_n) \|f_n y_n - p\|^2\} + (1 - a_n) \|f_n x_n - p\|^2$$

$$\|x_{n+1} - p\|^2 \leq a_n\{b_n(1 - z_n) \|x_n - p\|^2 + b_n z_n \|(T_n p - p T_n)(x_n - x_n - (T_n p - p))\| + b_n z \langle x - f_x, p - f_p \rangle^k + (1 - b_n) \|y_n - p\|^2\} + (1 - a_n) \|x_n - p\|^2$$

i.

$$\|x_{n+1} - p\|^2 \leq a_n\{b_n(1 - z_n) \|x_n - p\|^2 + (1 - b_n) \|y_n - p\|^2\} + (1 - a_n) \|x_n - p\|^2$$

$$\leq a_n\{b_n \|x_n - p\|^2 + (1 - b_n) \|x_n - p\|^2\} + (1 - a_n) \|x_n - p\|^2$$

$$\leq a_n \|x_n - p\|^2 + (1 - a_n) \|x_n - p\|^2 = \|x_n - p\|^2$$

By lemma (1.1), we get $\lim_{n \to \infty} \|x_n - p\|$ exists ,and hence $\langle f_n \rangle$are also bounded. So by lemma (1.4) we get $\langle P_C(x_n) \rangle$ converges strongly to the point in C.

$$\|x_n - T_n x_n\| \leq \|a_{n-1}(b_{n-1} T_{n-1} x_{n-1} + (1 - b_{n-1}) f_{n-1} y_{n-1} - T_n x_n) + (1 - a_{n-1}) f_{n-1} x_{n-1} - T_n x_n\|$$

$$\leq a_{n-1} \|b_{n-1} T_{n-1} x_{n-1} + (1 - b_{n-1}) T_n y_{n-1} - T_n x_n\| + b_{n-1} \|f_{n-1} x_{n-1} - T_n x_n\|$$

Since $\langle f_n \rangle$and $\langle T_n \rangle$ are also bounded and $\langle a_n \rangle, \langle b_n \rangle$are sequences in $(0,1]$ converges to 0. As $n \to \infty$, we get

$$\|x_n - T_n x_n\| \to 0 .$$

Now, since $\langle x_n \rangle$ is bounded then there exists subsequence $\langle x_{nk} \rangle$ of x_n such that $x_{nk} \rightharpoonup z$and $\|x_n - T_n x_n\| \to 0$. Then we get,

$z$is an asymptotic common fixed of T_n, for each $n \in N$.

Then the iteration process $\langle x_n \rangle$ has converges weakly to an asymptotic common fixed point of T_n , for each $n \in N$. ■

Now, we consider property $\mathcal{P}$ for any sequence as follows:

Let $\langle T_n \rangle$be a sequence of mappings we say that $\langle T_n \rangle$ has property if $\langle T_n \rangle$ satisfy the condition:

$$\|T_n - z\|^2 \leq \|T_n\|^2, \text{for each } z \in \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right).$$

In the following theorem we study the convergence for the iterationprocess

$$y_n = b'_n \left[a'_n x_n + (1 - a'_n) \sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n \right] + (1 - b'_n) g_n x_n$$

$$x_{n+1} = a'_n[a_n T_n x_n + b_n f_n x_n + c_n f_n g_n x_n] + b'_n[a_n b_n (T_n x_n - f_n x_n) + b_n c_n (f_n x_n - f_n T_n x_n) + c_n a_n (f_n T_n x_n - T_n x_n) + d_n g_n y_n] \quad (2.1)$$

where $\langle a'_n \rangle, \langle b'_n \rangle, \langle b_n \rangle, \langle c_n \rangle \langle a_n \rangle$and $\langle d_n \rangle$are sequences in $[0,1]$ such that $\langle a'_n \rangle, \langle b'_n \rangle$ converges to 0 , $a'_n \geq b'_n$. Such that

ii. $a'_n + b'_n = 1$, $a_n + b_n + c_n = 1$, $\sum_{i=1}^{m} \gamma_{n,i}$ and $d_n + a_n b_n + b_n c_n + c_n a_n = 1$

$\langle T_n - f_n \rangle, \langle f_n - f_n T_n \rangle, \langle f_n T_n - T_n \rangle$has property .

Theorem (2.5) :

Let $A_1, A_2, \ldots\ldots, A_m$ are maximal monotone multivalued mapping and $\emptyset \neq C$ convex closed in X, $\langle T_n \rangle$ is bounded sequences of expansion mappings on C and $\langle f_n \rangle, \langle g_n \rangle$ are sequences of nonexpansive mapping on C. If the iteration processdefined as (2.1) and $\left(Fix\left(J^i_{r_{n,i}}\right)\right) \cap (\cap_{n=1}^{\infty} Fix(T_n)) \cap (\cap_{n=1}^{\infty} Fix(f_n)) \cap (\cap_{n=1}^{\infty} Fix(g_n)) \neq \emptyset$.Then $\langle x_n \rangle$ has converges weakly to an asymptotic common fixed point of T_n, for each $n \in N$. Moreover $\langle P_C(x_n) \rangle$ converges strongly to the point in C.

Proof :

Let

$$p \in \left(Fix\left(J^i_{r_{n,i}}\right)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(g_n)\right)$$

$$\|y_n - p\|^2 \leq \left\| b'_n \left[\left(\sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n - p \right) \right] + \left[a'_n (x_n - p) + (1 - a'_n) \right] \right\|^2$$

$$\leq b'_n \left\| \left(\sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n - p \right) \right\|^2 + (1 - b'_n) \left\| a'_n (x_n - p) + (1 - a'_n) \right\|^2$$

$$\leq b'_n \left\| \left(\sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n - p \right) \right\|^2 + (1 - b'_n) \|P_c g_n x_n - p\|^2$$

$$\|y_n - p\|^2 \leq b'_n \left[\sum_{i=1}^{m} \gamma_{n,i} \left\| J^i_{r_{n,i}} x_n - p \right\|^2 \right] + (1 - b'_n) \|x_n - p\|^2$$

$$\leq b'_n [a'_n \|x_n - p\|^2 + (1 - a'_n) \|x_n - p\|^2] + (1 - b'_n) \|x_n - p\|^2$$

$$\|y_n - p\|^2 = b'_n \|x_n - p\|^2 + (1 - b'_n) \|x_n - p\|^2 = \|x_n - p\|^2$$

Hence, $\|y_n - p\|^2 \leq \|x_n - p\|^2$

Now, by (2.1), then we have

$$\|x_{n+1} - p\|^2 \leq a'_n \|a_n T_n x_n + b_n f_n x_n + c_n f_n g_n x_n - p\|^2 + b'_n \|a_n b_n (T_n x_n - f_n x_n) + b_n c_n (f_n x_n - f_n T_n x_n) + c_n a_n (f_n T_n x_n - T_n x_n) + d_n g_n y_n - p\|^2$$

$$\|x_{n+1} - p\|^2 \leq a'_n a_n \|T_n x_n - p\|^2 + a'_n b_n \|f_n x_n - p\|^2 + a'_n c_n \|f_n g_n x_n - p\|^2 - a'_n a_n b_n \|T_n x_n - f_n x_n\|^2 - a'_n b_n c_n \|f_n x_n - f_n T_n x_n\|^2 - a'_n c_n a_n \|f_n T_n x_n - T_n x_n\|^2 + b'_n a_n b_n \|T_n x_n - f_n x_n - p\|^2 + b'_n b_n c_n \|f_n x_n - f_n T_n x_n - p\|^2 + b'_n c_n a_n \|f_n T_n x_n - T_n x_n - p\|^2 - b_n d_n \|g_n y_n - p\|^2$$

$$\leq a'_n a_n \|T_n x_n - p\|^2 + a'_n b_n \|f_n x_n - p\|^2 + a'_n c_n \|f_n g_n x_n - p\|^2 - a'_n a_n b_n \|T_n x_n - f_n x_n\| - a'_n b_n c_n \|f_n x_n - f_n T_n x_n\|^2 - a'_n c_n a_n \|f_n T_n x_n - T_n x_n\|^2 + a'_n a_n b_n \|T_n x_n - f_n x_n\|^2 + a'_n b_n c_n \|f_n x_n - f_n T_n x_n\|^2 + a'_n c_n a_n \|f_n T_n x_n - T_n x_n\|^2 - b_n d_n \|g_n y_n - p\|^2$$

for each sequence $\langle z_n \rangle$ in $[0,1]$ converges to 0 there exists anon – negative real numberz such that

$$\|x_{n+1} - p\|^2 \leq a'_n a_n [(1 - z_n) \|x_n - p\|^2 + z_n \|p - T_n p\| . \|(p - T_n p)(_c x_n - T_n x_n - (p - T_n p))\| + z(\langle p - T_n p, x_n - T_n x_n \rangle)] + a'_n b_n \|x_n - p\|^2 + a'_n c_n \|x_n - p\|^2 + b'_n d_n \|y_n - p\|^2$$

Now,

$$\|x_{n+1} - p\|^2 \leq a'_n a_n \|x_n - p\|^2 + a'_n b_n \|x_n - p\|^2 + a'_n c_n \|x_n - p\|^2 + b'_n d_n \|x_n - p\|^2$$

$$\|x_{n+1} - p\|^2 = a'_n \|x_n - p\|^2 + (1 - a'_n) \|x_n - p\|^2 = \|x_n - p\|^2$$

By lemma (1.1),

we get $\lim_{n \to \infty} \|x_n - p\|$ exists . Hence, $\langle x_n \rangle$ is bounded sequence , so that $\langle g_n \rangle$, and $\langle f_n \rangle$ are also bounded sequences

So, by lemma (1.4) we deduce $\langle P_C(x_n) \rangle$ converges strongly to the point in C.

$$\|x_n - T_n x_n\| = \|a'_{n-1} [a_{n-1} T_{n-1} x_{n-1} + b_{n-1} f_{n-1} x_{n-1} + c_{n-1} f_{n-1} g_{n-1} x_{n-1}] + b'_{n-1} [a_{n-1} b_{n-1} (T_{n-1} x_{n-1} - f_{n-1} x_{n-1}) + b_{n-1} c_{n-1} (f_{n-1} x_{n-1} - f_{n-1} T_{n-1} x_{n-1}) + c_{n-1} a_{n-1} (f_{n-1} T_{n-1} x_{n-1} - T_{n-1} x_{n-1}) + d_{n-1} g_{n-1} y_{n-1}] - T_n y_n\|$$

$$\|x_n - T_n x_n\| \le a'_{n-1}\|a_{n-1}T_{n-1}x_{n-1} + b_{n-1}f_{n-1}x_{n-1} + c_{n-1}f_{n-1}T_{n-1}x_{n-1} - g_n y_n\| + b'_{n-1}\|a_{n-1}b_{n-1}(T_{n-1}x_{n-1} - f_{n-1}x_{n-1}) + b_{n-1}c_{n-1}(f_{n-1}x_{n-1} - f_{n-1}T_{n-1}x_{n-1}) + c_{n-1}\alpha_{n-1}(f_{n-1}T_{n-1}x_{n-1} - T_{n-1}x_{n-1}) + d_{n-1}g_{n-1}y_{n-1} - T_n y_n\|$$

Since $\acute{a}_n, \acute{b}_n \to 0$ and $\langle T_n\rangle, \langle f_n\rangle$ and $\langle g_n\rangle$ are bounded ,then we get

$$\|x_n - T_n x_n\| \to 0 \text{ as } n \to \infty$$

Now, since $\langle x_n\rangle$ is bounded sequence then there exists subsequence $\langle x_{nk}\rangle$ of $\langle x_n\rangle$ such that $x_{nk} \rightharpoonup z$ and since $\|x_n - T_n x_n\| \to 0$,then we get,

zis asymptotic common fixed point of T_n, for all $n \in N$.

Then the iteration process $\langle x_n\rangle$ has converges weakly to an asymptotic common fixed point of T_n, for all $n \in N$. ■

In the following theorem we give a new iteration processand we study the convergence for this iteration to an asymptotic common fixed point.

Theorem (2.6) :

If $\langle f_n\rangle$ be a sequence of nonexpansive mapping on C and $\langle T_n\rangle$ be a bounded sequence of expansion mappings on C.Define the proximal point scheme $\langle x_n\rangle$ as follows:

$$y_n = a'_n f_n x_n + (1 - a'_n)\left(b'_n T_n x_n + (1 - b'_n) f_n T_n x_n\right)$$

$$x_{n+1} = b_n\left[a_n \sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n + (1 - a_n) f_n x_n\right] + (1 - b_n) g_n f_n y_n \quad (2.2)$$

where $\langle a'_n\rangle, \langle b'_n\rangle, \langle a_n\rangle, \langle b_n\rangle$ are sequences in $[0,1]$such that $\langle a_n\rangle, \langle b_n\rangle$ converges to 0 such that $a_n + b_n \le 1$. If $\left(\bigcap_{n=1}^{\infty} Fix\left(J^i_{r_{n,i}}\right)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right) \neq \emptyset$. Then the iteration process $\langle x_n\rangle$ has converges weakly to an asymptotic common fixed point of T_n , for all $n \in N$. Moreover $\langle P_C(x_n)\rangle$ converges strongly to the point in C.

Proof :

Let

$p \in \left(Fix(P_c)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(T_n)\right) \cap \left(\bigcap_{n=1}^{\infty} Fix(f_n)\right)$

Since $y_n = a'_n f_n x_n + (1 - a'_n)\left(b'_n P_c T_n x_n + (1 - b'_n) f_n P_c T_n x_n\right)$ then we have,

$$\|y_n - p\|^2 \le \acute{a}\|f_n x_n - p\|^2 + (1 - a'_n)\left[b'_n\|T_n x_n - p\|^2 + (1 - b'_n)\|f_n T_n x_n - p\|^2\right]$$

$$\|y_n - p\|^2 \le a'_n\|x_n - p\|^2 + (1 - a'_n)\left[b'_n\|T_n x_n - p\|^2 + (1 - b'_n)\|T_n x_n - p\|^2\right] \le a'_n\|x_n - p\|^2 + (1 - a'_n)\left[b'_n\|T_n x_n - p\|^2 + (1 - b'_n)\|T_n x_n - p\|^2\right] = a'_n\|x_n - p\|^2 + (1 - a'_n)\|T_n x_n - p\|^2$$

for each sequence $\langle z_n\rangle$ in $[0,1]$ converges to 0 there exists anon – negative real numberz such that

$$\|y_n - p\|^2 \le a'_n\|x_n - p\|^2 + (1 - a'_n)\left[(1 - z_n)\|x_n - p\|^2 + z_n.\|(p - T_n p)(x_n - T_n x_n - (p - T_n p))\| + z(\langle x_n - T_n x_n, p - T_n p\rangle)^k\right]$$

$$\|y_n - p\|^2 \le \acute{a}\|x_n - p\|^2 + (1 - a'_n)\|x_n - p\|^2 = \|x_n - p\|^2$$

Hence, $\|y_n - p\|^2 \le \|x_n - p\|^2$

$$b_n\left[a_n \sum_{i=1}^{m} \gamma_{n,i} + (1 - a_n) f_n x_n\right] + (1 - b_n) g_n f_n y_n$$

$$\|x_{n+1} - p\|^2 \le b_n\left[a_n \sum_{i=1}^{m} \gamma_{n,i} \left\|J^i_{r_{n,i}} x_n - p\right\|^2 + (1 - a_n)\|f_n x_n - p\|^2\right] + (1 - b_n)\|g_n f_n y_n - p\|^2$$

$$\|x_{n+1} - p\|^2 \le b_n\left[a_n \sum_{i=1}^{m} \gamma_{n,i} \|x_n - p\|^2 + (1 - a_n)\|x_n - p\|^2\right] + (1 - b_n)\|f_n y_n - p\|^2$$

$$\|x_{n+1} - p\|^2 \le\le b_n[a_n\|x_n - p\|^2 + (1 - a_n)\|x_n - p\|^2] + (1 - b_n)\|y_n - p\|^2 \le b_n\|x_n - p\|^2 + (1 - b_n)\|x_n - p\|^2 = \|x_n - p\|^2$$

By lemma (1.1), we get $\lim_{n\to\infty}\|x_n - p\|$exists

Hence the iteration $\langle x_n\rangle$ is bounded sequence. So $\langle f_n\rangle$ and $\langle g_n\rangle$ also bounded sequences So, by lemma (1.4) we deduce $\langle P_C(x_n)\rangle$ converges strongly to the point in C. Now,

$$\|x_n - T_n x_n\| = \left\|b_{n-1}\left[a_{n-1} \sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n + (1 - a_{n-1}) f_n x_{n-1}\right] + (1 - b_{n-1}) g_{n-1} f_{n-`1} y_{n-1} - T_n x_n\right\|$$

$$\|x_n - T_n x_n\| \le b_{n-1} \Big\| a_{n-1} \sum_{i=1}^{m} \gamma_{n,i} J^i_{r_{n,i}} x_n + (1 - a_{n-1}) f_n x_{n-1} - T_n x_n \Big\| + a_{n-1} \|g_{n-1} f_{n-1} P_c y_{n-1} - T_n x_n\|$$

As, $n \to \infty$ we get $\|x_n - T_n x_n\| \to 0$

Since $\langle x_n \rangle$ bounded sequencethen there exist $\langle x_{nk} \rangle$ subsequence of $\langle x_n \rangle$ such that $x_{nk} \rightharpoonup z$. Since $\|x_n - T_n x_n\| \to 0$, Then we get,

zis an asymptotic common fixed of T_n, for each $n \in N$. And hence,

the iteration process$\langle x_n \rangle$has converges weakly to an asymptotic common fixed point of T_n, for each $n \in N$. ■

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