RESPONSE SURFACE METHODOLOGY MODELING AND OPTIMIZATION OF REINFORCEMENT FRAME DEFLECTION AND COST

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Abstract - A rectangular frame was reinforced by using rods. The rods were arranged in a number of different ways, so that they crossed the frame in a vertical, horizontal and inclined manner. Some permutations involved rods of different arrangements together. Investigations were established to reflect a central composite design in response surface methodology. The frame free edge had a theoretical deflection and this was calculated. Introducing rods into the frame allowed the torsional resistance to change for the various arrangements and this gave rise to different deflections. When the rods were arranged so that they crossed the frame or traversed it, the torsional resistance was highest. Design of experiments (DOE) calculations within the Minitab program allowed the results of these investigations to be analyzed and the deflection to be minimized. Input factors and their interactions were also analyzed along with their corresponding effects. In addition to the overall effect on the deflection, the cost of the rods was also considered. The deflection was most significantly decreased with the use of longitudinal rods, whilst cost was most minimized with the use of transverse rods. It was clear that the model was able to predict the deflection because the model had a high correlation coefficient of R^2 = 97.991% with the experimental data.

Keywords - Frame deflection, frame torsional resistance, frame stiffening, design of experiment (DOE), Response surface method (RSM).

I. INTRODUCTION

When a force is applied to a body so that it twists about its structural axis, this is referred to as torsional loading. Examples of torsional loading are commonplace in systems such as axles, spindles and other rotating frame bodies. The principal deflection caused by torsion is measured by the angle of twist, or by the vertical movement of one corner of the frame or base [7] [10]. Stiffeners can be welded to the frame so its stiffness increases. These rods can be arranged in a transverse, longitudinal or inclined fashion, and this will reduce the vertical deflection[8].. The optimum design of a set of compressed plates on a single axis, with rods across them in a plethora of different arrangements can be achieved using two distinct mechanisms. The results are then compared. Initial imperfections and residual welding stresses are built into the calculations of both methods, but in different ways. Paik's solution is used to calculate the elastic deflection as a result of both compressive and lateral forces. There are also further deformations that occur along with this deflection, and these are a result of lateral pressure and the longitudinal welds causing shrinkage to occur [16].There is a need to simplify the design method for beams on semi-rigid composite frames that are loaded vertically at the serviceability limit state. As well as that, the deflections of the composite beam may be calculated from equations. As a result, both joint flexibility and beam sectional properties can be derived, and the connection secant stiffness also has a formula. The experimental results for two semi-rigid composite frames validates the equations and ensures that they agree well with the experimental data. This is mainly because they make allowances for the actual beam-to-column connection [13]. An infinite series approach then allows thin rectangular plates loaded at various points by point loads and stiffened in various ways by elastic beams to be determined. The formulae refer to bending as well as torsional beam stiffness. When results are obtained, they are manipulated for the different plate aspect ratios and beam stiffness values. The available solutions provide results that are consistent, and parametric studies show the utility of this approach for various ratios of beam stiffness (bending and torsional) to plate stiffness [4]. The aim of this work is the deflection optimization of reinforcement frame using various transverse and longitudinal bars with the aid of DOE method.

II. THEORETICAL DEFLECTION ANALYSIS OF A BUILT-UP FRAME

When the strength of a built-up frame is analyzed, along with its resistance to twisting, the applied load is considered as two forces forming of a couple at each end of the frame. These forces apply the transverse and longitudinal torque of the frame. This indicates that the longitudinal and transverse member resistance make up the overall resistance to twisting. It is usually more convenient to express the resulting angular twists in term of vertical deflection of the frame corner at which the vertical load acts [8].

A vertical force (F) applied at the corner of the frame will cause it to deflect by a vertical distance (Δ). The longitudinal and transverse member twists by an angle Δ_L and Δ_T respectively when this happens, allowing the following equation to be formed:

$$\Delta = \theta_L W = \theta_T L \qquad (1)$$

So:

$$\theta_L = \frac{\Delta}{W} \quad and \quad \theta_T = \frac{\Delta}{L} \qquad (2)$$

Where:

Δ: vertical deflection under the applied load (mm)
Δ_L: longitudinal twist angle (rads)
Δ_T: transverse twist angle (rads)
L: length of longitudinal member (mm)
W: length of transverse member (mm)

For a flat member, the torsional resistance (*R*) gives a greater accuracy than the polar moment of inertia[8] therefore the twist angle (θ) can be written as follows:

$$\theta = \frac{TL}{RG}$$

Using the common formula for angular twist:

$$\theta_L = \frac{T_L L}{G n_L R_L} \quad and \quad \theta_T = \frac{T_T W}{G n_T R_T} \qquad (3)$$

Where:

T_L&T_T = the applied torque at the longitudinal and transverse member (N.mm)
n_L&n_T = the number of longitudinal and transverse members
G = the shear modulus (N.mm^{-2})
R_L& R_T = the torsional resistance of longitudinal and transverse members (mm^4)

In this work, the cross sectional area of longitudinal and transverse members is the same and has a square area of 2mm x 2mm = 4mm$^{2.}$.

The torsional resistance for this section will be[8].

$R = R_L = R_T = 0.1406a^4$

Substituting equation (3) in (2) give:

$$\frac{\Delta}{W} = \frac{T_L L}{G n_L R_L} \quad and \quad \frac{\Delta}{L} = \frac{T_T W}{G n_T R_T} \qquad (4)$$

Then:

$$T_L = \frac{\Delta G n_L R_L}{LW} \quad and \quad T_T = \frac{\Delta G n_T R_T}{LW} \qquad (5)$$

Since the applied torque is:

$$T_L = P_L W \quad and \quad T_T = P_T L \qquad (6)$$

$$\therefore P_L = \frac{T_L}{W} \quad and \quad P_T = \frac{T_T}{L} \qquad (7)$$

Substituting Eq.(7) in Eq.(5) gives:

$$P_L = \frac{\Delta G n_L R_L}{LW^2} \qquad and \qquad P_T = \frac{\Delta G n_T R_T}{WL^2}$$

Since the external force (F) applied at the corner of frame is the sum of these two forces:

$$F = P_L + P_T = \frac{\Delta G n_L R_L}{LW^2} + \frac{\Delta G n_T R_T}{WL^2} = \frac{\Delta G}{WL}\left(\frac{n_L R_L}{W} + \frac{n_T R_T}{L}\right) \qquad (8)$$

or:

$$\Delta = \frac{FWL}{G}\left(\frac{1}{\left(\frac{n_L R_L}{W} + \frac{n_T R_T}{L}\right)}\right) \qquad (9)$$

Eq.(9) represents the theoretical vertical deflection of the free end edge.

III. THE STUDIED CASES

Figure 1 shows how the free end of the frame is deflected and this is used as a reference deflection under an applied load of 1N which is used to compare the deflections of other frames.

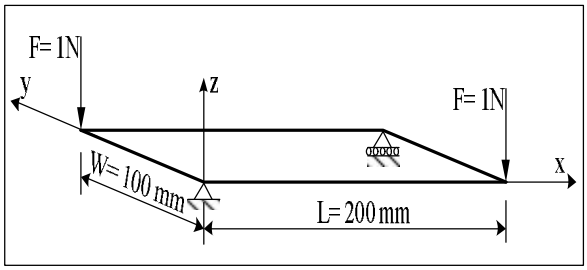
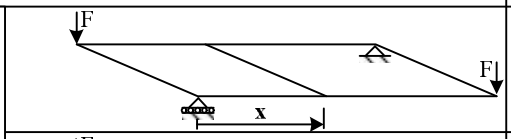
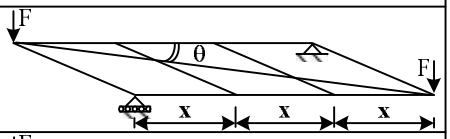
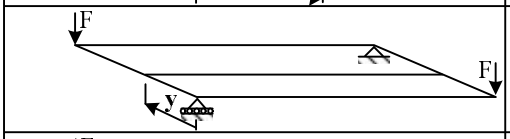
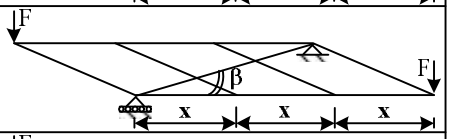
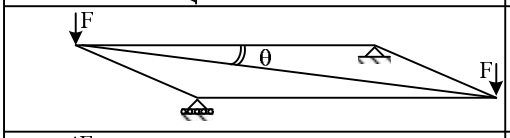
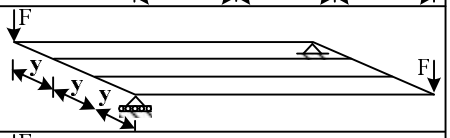
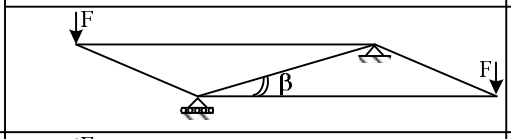
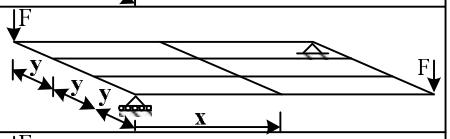
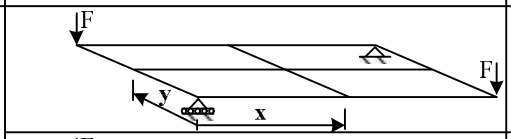
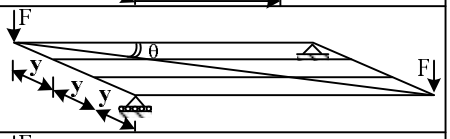
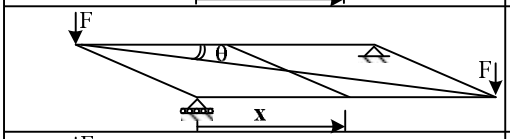
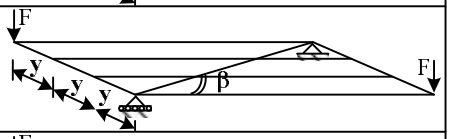
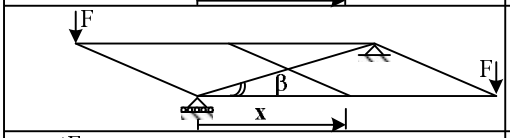
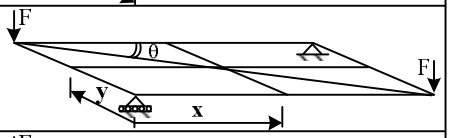
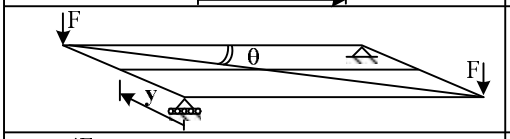
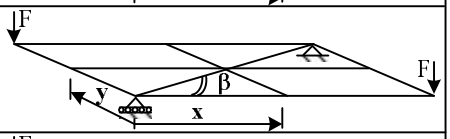
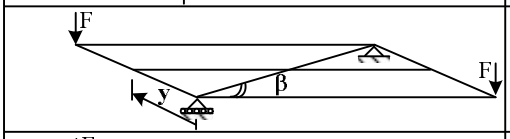
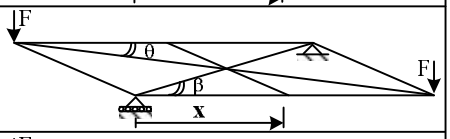
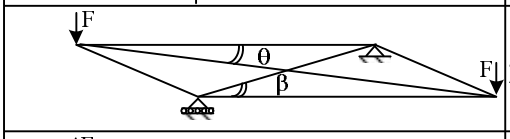
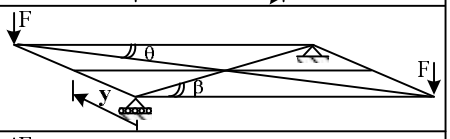
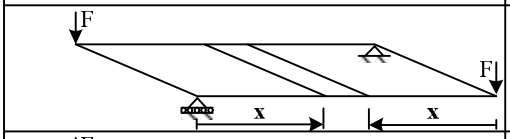
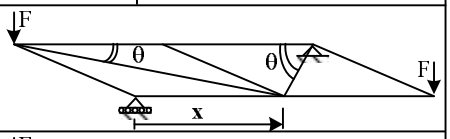
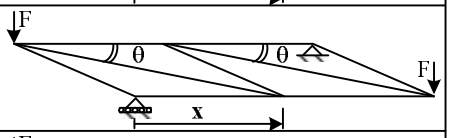
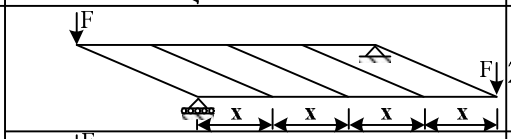
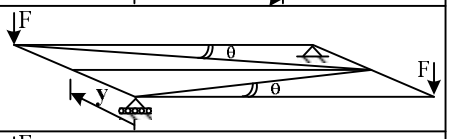
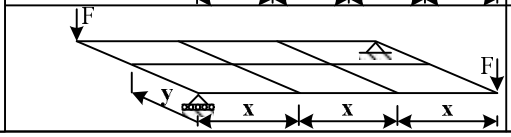
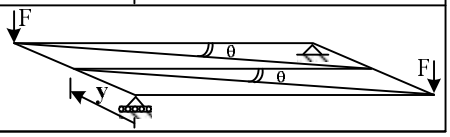


Figure 1 Schematic of simply supported frame at the opposite corner.

The frame consists of four rods; two in the longitudinal direction, 200mm length, and two in the transverse direction with a length of 100mm each rod has a square cross sectional area of 4 mm^2.

The frame set out above has up to three additional rods attached to it. This serves to strengthen the frame and minimize its deflection when a load is applied to it. These rods are identical to the frame members and have a cross-sectional area of 4mm^2. Table 1 sets out the various permutations in which the rods are added.
A single rod may be added to the frame in four different ways as shown in specimen groups 1-4 of set A, in Table 1. Two rods may be added in eight different ways as shown in specimen groups 5-12 of set B. Three rods may be added in sixteen different ways as shown in specimens group 13- 28 of set C. Table 2 lists the coordinates (x & y) as well as the inclined angles (θ &β).

Table (1) all samples of Case Studied

No.	Case Studied		No.	Case Studied	
1	Set A		15	Set C	
2			16		
3			17		
4			18		
5	Set B		19		
6			20		
7			21		
8			22		
9			23		
10			24		
11			25		
12			26		
13	Set C		27		
14			28		

No.	Coordinate	No.	Coordinate
1	X = 2, 5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 90, 99 & 101 mm.	15	X=69 mm & θ =63.5
2	Y = 5, 10, 15, 20, 25, 30, 35, 40, 45 & 49 mm.	16	X=69 mm & β =63.5^o
3	θ = 10, 20, 30, 40, 50, 60, 63.5, 70 & 80°	17	Y =30 mm
4	β = 10, 20, 30, 40, 50, 60, 63.5, 70 & 80°	18	Y =30 mm & x =101 mm.
5	X =101 mm & y=49	19	Y =30 mm & θ =63.5^o
6	X =101 mm & θ=63.5^o	20	y=30 mm & β =63.5 ^{o}
7	X =101 mm &β =63.5 ^{o}	21	X =101 mm, y =49 mm & θ =63.5^o
8	y=49 mm & β =63.5 ^{o}	22	X =101 mm, y =49 mm & β =63.5^o
9	y=49 mm & θ =63.5 ^{o}	23	X =101 mm, θ=63.5^o& β=63.5^o
10	β =63.5 ^{o} & θ =63.5 ^{o}	24	Y =101 mm, θ=63.5^o& β=63.5^o
11	X=2,5,10,15,20,30,40,50,60,70,80,90 &99 mm	25	X =101 mm & θ=26°
12	Y=5,10,15,20,25,30,35,40 &45 mm	26	X =101 mm & θ =26°
13	X =67 mm	27	Y =101 mm & θ =13.5^o
14	X =67 mm & y =49 mm	28	X =101 mm & θ =13.5^o

Table 2. The coordinates (x & y) and the inclined angles(θ&β)

RESULTS

The results data presents the deflection of the free end of the frame under the applied load for each sample shown in Table 1. The deflection of the basic frame shape shown in Figure 2 is calculated using theoretical Eq.9 and compared with the prediction using the response surface model. Hence, the exact solution of Eq.9 is 1.93 mm, which has a very low error value compared to the predicted value of 1.899 mm. Table 3 sets out these results.

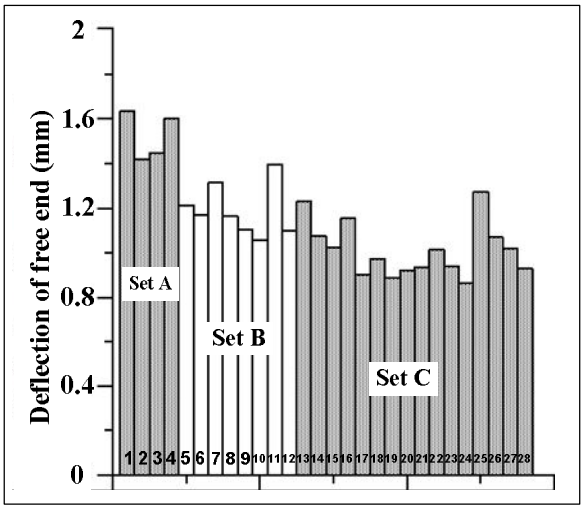


Figure 2 The deflection of free end samples for sets A, B and C

Theoretical	Prediction Using response surface model
Deflection (mm) = 1.93	Deflection (mm) = 1.899

Table 3. Deflection of basic frame shape

In order to compare the deflection of all samples, the minimum deflection for each of the samples from Table 1 is plotted in Figure 2.

SIGN OF EXPERIMENT (DOE) USING RESPONE SURFACE METHOD

The input factors and important interactions that have a significant impact on response are assessed using design of experiment. After identifying the significant factors and interactions, the experimental runs are optimized. Further optimization can then allow the important input settings to achieve the desired output [6][9].The input variables and the response are both present in the design matrix used in the fractional DOE.

A Minitab program is used to input and analyze the data [11] for input variables, coded units are used, where +1 represents the maximum value in the sample and -1 represents the minimum value. Table 4 defines the input variables used in Minitab program:

Variable	Definition (from Table 1)
X1	One rod as in set(A), No1 with (x=101 mm)
X2	Two rods as in set(B), No11 with (x= 66.66 mm)
X4	Three rods as in set(C), No13 with (x= 66.66 mm)
Y1	One rod as in set(A), No2 with (y=49 mm)
Y2	Two rods as in set(B), No12 with (y= 33.33 mm)
Y3	Three rods as in set(C), No17 with (y=33.33 mm)
T	One rod as in set(A), No3 with (θ= 63.5 deg.)
B	One rod as in set(A), No4 with (β=63.5 deg.)
V	One rod as in set(C), No26 with (θ= 45 deg.)
W	One rod as in set(C), No27 with (θ= 26.5 deg.)

Table 4. The input variable in Minitab program

All the different arrangements shown in Table 1 are analyzed in Table 4. The response in the Minitab program considers both the deflection and the cost of stiffeners of the frame. The main objective is to minimize the deflection whilst also minimizing the cost. Since the first response is deflection, all the calculated deflection values are input into the project window of Minitab. The second response is cost, which is assumed to be 1 unit for each stiffener of 100 mm length. This allows the cost of the various permutations of the frame stiffeners to be calculated. By analyzing the experiments the factors that affect responses and the important interactions can be identified.

Analysis of variance (ANOVA) was used to model the response and their corresponding parameters. These responses well also optimized. The statistical parameters were estimated using pareto plots by means of response surface methodology (RSM). If all variables are assumed to be measurable, the response surface can be expressed as follows[9]:

$$Y = f(X_1, X_2, X_3, X_4 \ldots\ldots, X_n) \qquad (11)$$

Where Y is the response of the system and X_i is the variables of action called factors. The target of the RSM is to optimize the response variable (Y) and to find a suitable estimate to the functional relationship between the independent variables and the response surface. A quadratic equation was used for model formation. Applying the relationships in Table 2, the values of the codes were calculated and shown in Table3.Where X_{max} and $X_{min}$are maximum and minimum values of X respectively β is $2^{n/4}$, n= number of variables [15]. In this studyβ=$2^{3/4}$=1.682

$$Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \left(\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij}\ x_i x_j \right)_{i<j} \qquad (12)$$

Here, Y is the predicted response evaluated,x_i and x_j are the variables,$\beta_0$is the constant coefficient,β_i, β_{ii} and $\beta_{ij}$are the inter-action coefficients of linear, quadratic, and the second order terms respectively, and *k* is the number of studied factors. For statistical analysis, the experimental variables X_i have been coded as x_i according to the following equation.

$$x_i = \frac{(X_i - X_n)}{\Delta X_i} \qquad (13)$$

where x_i is the coded value (dimensionless) of the ith independent variable, X_i is the uncoded value of the ith independent variable, X_n is the value of X_i at the center point, and ΔX_i is the step change value of the real variable i. From Equation 13, the coded unit can be converted into an uncoded unit. In a central composite design, the coded values of -1 and +1 of alpha are the low and high values of the variables respectively.

THE PARETO CHART OF THE STANDRDIZED EFFECTS

The generated Pareto chart presents the significance of the terms. The significance of any of the bars can be identified when they are plotted past the red line. Figure 3 shows the Pareto chart for the first response (deflection). There are no more three- way interactions. Most of the two-way interactions are not significant so they may be removed from this chart. The factors that are showing as significant are shown in Figure 4 .From this chart, it can be seen that the main variable that has an effect on the deflection behavior isG which represent the term T in Table 4. One rod as in set (A), No.3 with (θ= 63.5deg.)

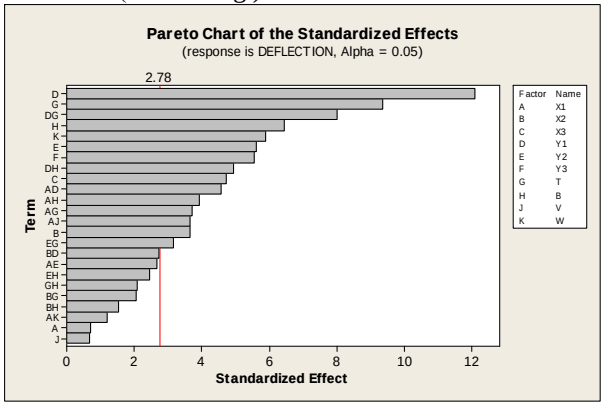


Figure 3 Deflection Pareto chart

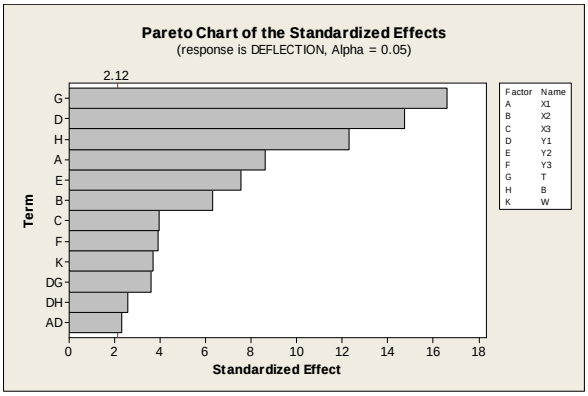


Figure 4 Deflection Pareto chart (reviser)

The Pareto chart for cost is shown in Figure 5. This chart shows that the maximum and minimum effect appears in variables H and B which represent the terms B and X2 in Table 4.

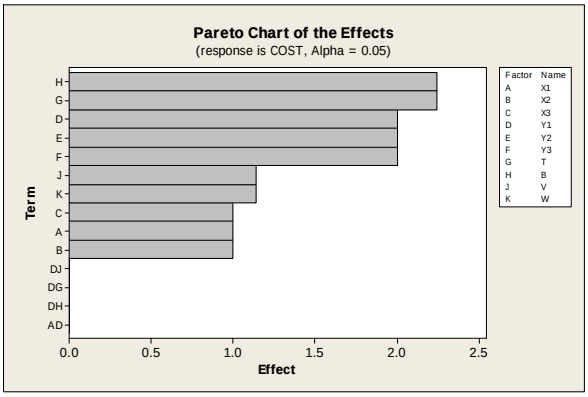


Figure 5 Cost Pareto chart

MAIN EFFECT PLOT

The main objective of this work is to minimize the deflection by using stiffeners and the effects of this can be seen in Figure 6. Since the units of the vertical axes are the same, the effects of the different variables may be compared. The plot with the steepest line has the greatest effect on the deflection response. The existence of variables X1 and X2 has a minimal effect on the deflection response whereas the existence of variables Y1,Y2 and Y3 caused the deflection to be significantly reduced.

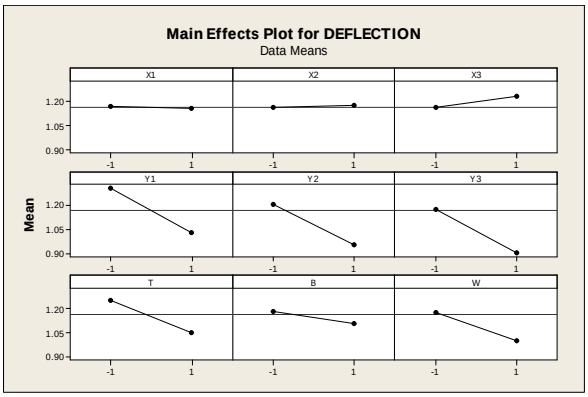


Figure 6 Main effect plot for deflection

The main effect plot for cost, Figure 7, illustrates that the reduction in cost occurs with the existence of variables X1, X2 and X3. All the other variables cause an increase in cost.

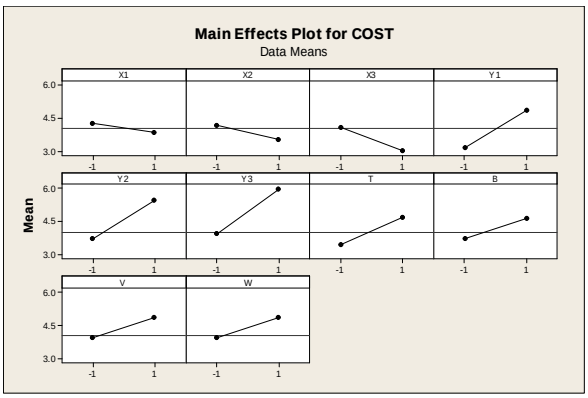


Figure 7 Main effect plot for cost

INTERACTION PLOT FOR DEFLECTION

The interaction induced between the variables X1, Y1, T and B are shown in Figure 8. The greatest impact of the interaction variables occurs between X1 and Y1. This interaction gives the highest reduction in deflection. The other variables interact significantly. This can be judged from interaction plots, which have a relatively low gradient. The response optimizer allows the deflection with the minimum cost to be found. The minimum deflection is 0.8644mm whilst the minimum cost is 5 units. This occurs with the existence of the variable X1, X2, X3, and Y1 but in the absence of the other variables, as seen in Figure 9. This optimization has been achieved with first priority given to deflection and the second to cost.

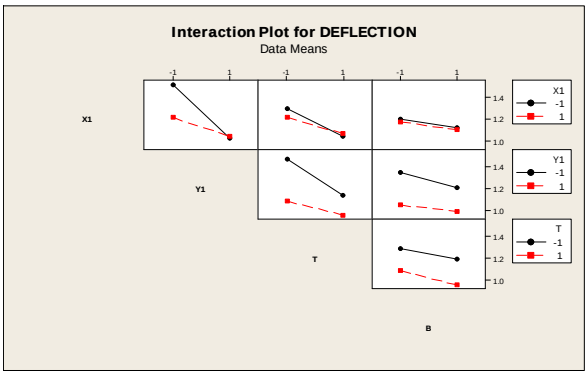


Figure 8 Interaction plot for deflection

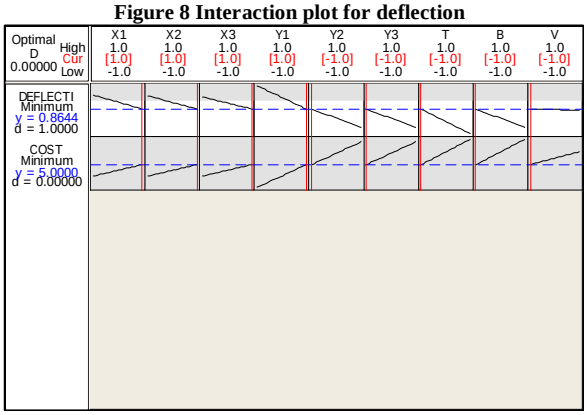


Figure 9 Response optimizer

DISCUSSION

The most significant reduction in deflection is obtained when the stiffener rod (or rods) divide the longitudinal or transverse length of frame into equal spans or when the bar ends are attached at the corner of the frame. As a result, all the samples were tested according to those conditions.

For set B, the minimum deflection is observed in sample (10), for which arrangement the stiffener bars were arranged so that they are crossing the frame. On the other hand, the minimum global frame torsional resistance, and therefore maximum deflection, is observed in sample (11). In this case, the stiffener bars are arranged in the transverse direction with an equal span.

For set C, the maximum global torsional resistance is observed in sample (24), for which arrangement the first two stiffener bars were crossed each other whilst the third was situated in the longitudinal direction. The weakest case was observed in sample (25). In this case, the first two stiffener bars were connected in a V arrangement whilst the third bar was situated in the transverse direction.

CONCLUSION

1. The global torsional resistance is reduced uniformly when the reinforcing rods are welded on in the longitudinal and transverse direction.
2. When the reinforcing rods are arranged in a cross and/or longitudinal arrangements, the torsional resistance increases to its maximum value.
3. Transverse rods have a lower effect on reducing the deflection in comparison with longitudinal rods.
4. A minimum deflection is achieved by connecting the rods in a cross arrangement.
5. The minimum global frame torsional resistance is observed when the reinforcing rods are arranged in the transverse direction with equal span, the global frame torsional resistance is reduced to its minimum value.
6. In addition to the arrangement of the rods, the torsional stiffness of a reinforced frame also relies on the distribution of the stiffener rods.
7. The introduction of an inclined rod with ($\theta = 63.5^{o}$) caused the most significant effect on the deflection of the frame.
8. The introduction of the inclined bar with ($\beta = 63.5^{o}$) caused the most significant effect on the cost.
9. Only the introduction of transverse bars caused the cost to be decreased.
10. The response surface method gave as a good indication about the modeling and optimization of theoretical deflection.

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