A FUZZY APPROACH FOR DISCOVERY OF WEB USAGE PATTERNS FROM WEB LOG DATA

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Abstract - Web server access logs contain substantial data about the accesses of users to a Web site. In order to reveal the information about user preferences from, Web Usage Mining (WUM) is being performed. WUM contains three main steps: preprocessing, knowledge extraction and results analysis. During the preprocessing stage, raw web log data is transformed into a set of user profiles. Each user profile captures a set of URLs representing a user session. Clustering can be applied to this sessionized data in order to capture similar interests and trends among users' navigational patterns. Since the sessionized data may contain thousands of user sessions and each user session may consist of hundreds of URL accesses, dimensionality reduction is achieved by eliminating the low support URLs. But direct elimination of low support URLs and small sized sessions may results in loss of a significant amount of information especially when the count of low support URLs and small sessions is large. We propose a fuzzy solution to deal with this problem by assigning weights to URLs and user sessions based on a fuzzy membership function. After assigning the weights we apply a Fuzzy c-Mean Clustering algorithm to discover the clusters of user profiles. Our results show that fuzzy feature evaluation and dimensionality reduction results in better performance and validity indices for the discovered clusters.

Keywords - Web usage mining; fuzzy c-means clustering, feature evaluation; dimensionality reduction.

I. INTRODUCTION

The World Wide Web as a large and dynamic information source is a fertile ground for data mining principles or Web Mining. Web mining is primarily aimed at deriving actionable knowledge from the Web through the application of various data mining techniques [1]. Web Usage Mining is the discovery of user access patterns from Web server access logs [2]. Web Usage Mining (WUM) consists of three main steps: preprocessing, knowledge extraction and results analysis [3] [4]. The goal of the preprocessing step is to transform the raw web log data into a set of user profiles. Each such profile captures a sequence or a set of URLs representing a user session [5]. Web usage data preprocessing exploit a variety of algorithms and heuristic techniques for various preprocessing tasks such as data cleaning, user identification, user session identification etc [6]. Data cleaning involves tasks such as, removing extraneous references to embedded objects, style files, graphics, or sound files, and removing references due to spider navigations [7]. User identification refers to the process of identifying unique users from the user activity logs [8]. User Session identification is the process of segmenting the user activity log of each user into sessions, each representing a single visit to the site [9]. If the data mining task at hand is clustering, the session files are filtered to remove very small sessions in order to eliminate the noise from the data [10]. But direct removal of these small sized sessions may result in loss of a significant amount of information especially when the number of small sessions is large. We propose a "Fuzzy Set Theoretic" approach for filtering the low support URLs. We assign weights to all the URLs using a "Fuzzy Membership Function" based on their user sessions support count. After assigning the weights to URLs and User Sessions, we apply a "Fuzzy c-Mean Clustering" algorithm [11] to discover the clusters of user profiles [12]. Rest of the paper is organized as follows: in section-II, we provide a detailed description of our fuzzy set theoretic approach for feature evaluation and dimensionality reduction. In section III, we discuss how to apply Fuzzy c-Mean Clustering algorithm to the weighted as well as non-weighted sessionized data. Section IV discusses the results. Finally section V provides a brief conclusion.

II. FEATURE SELECTION AND SESSION WEIGHT ASSIGNMENT

We map the user sessions as vectors of URL references in a n-dimensional space. Let $U = \{u_1, u_2, \ldots, u_n\}$ be a set of n unique URLs appearing in the pre processed log and let $S = \{s_1, s_2, \ldots, s_m\}$ be a set of m user sessions discovered by preprocessing the web log data, where each user session $s_i \in S$ can be represented as $s = \{w_{u_1}, w_{u_2}, \ldots, w_{u_m}\}$. Each w_{u_i} may be either a binary or non-binary value depending on whether it represents presence and absence of the URL in the session or some other feature of the URL. If w_{u_i} represents presence of absence of the URL in the session, then each user session is represented as a bit vector where

$$w_{u_i} = \begin{cases} 1; \text{ if } u_i \in s; \\ 0; \text{ otherwise} \end{cases} \quad (1)$$

Instead of binary weights, feature weights can also be used to represent a user session. These feature weights may be based on frequency of occurrence of a URL reference within the user session, the time a user spends on a particular page or the number of bytes downloaded by the user from a page.

A. Fuzzy Feature Evaluation and Dimensionality Reduction by Assigning Weights to the URL Items:

The URLs appearing in the access logs could number in the thousands. Distance-based clustering methods often perform very poor when dealing with very high dimensional data. Therefore filtering the logs by removing references to low support URLs (i.e. that are not supported by a specified number of user sessions) can provide an effective dimensionality reduction method while improving clustering results. We propose a "Fuzzy Set Theoretic" approach for filtering the low support URLs. We assign weights to all the URLs using a "Fuzzy Membership Function" based on their user sessions support count. In this approach we use lower and upper bounds for the user session support. All the URLs with session support count lower than the lower bound α_1 are assigned the weight 0. On the all the URLs with session support count higher than the upper bound α_2 are assigned the weight 1. All the URLs with session support count between α_1 and α_2 are assigned the weight between 0 and 1 using a linear fuzzy membership function.

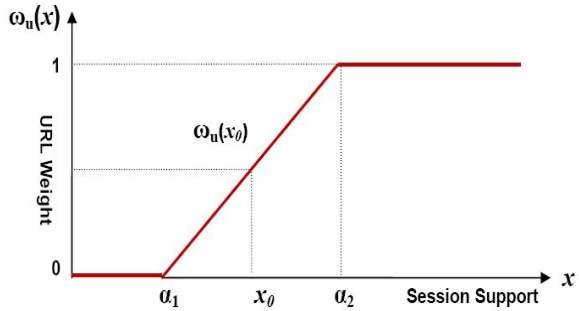

Fig. 1. Linear Fuzzy Membership Function for URL Weight Assignment

Fig. 1 depicts a linear fuzzy membership function for URL weight assignment. The URL weight assignment using the linear fuzzy membership function is carried out using the following formula:

$$\omega_u(x) = \begin{cases} 0, \text{if } x \le \alpha_1 \\ 1, \text{if } x \ge \alpha_2 \\ \dfrac{x-\alpha_1}{\alpha_2-\alpha_1}, \text{otherwise} \end{cases} \quad (2)$$

where

ω_u is weight assigned to the URL u

α_1 is the lower threshold on the session support count

α_2 is the upper threshold on the session support count

x is the session support count of URL u

B. Fuzzy Approach for Assigning Weights to User Sessions:

Before performing the clustering of the user sessions, the session files can be filtered to remove very small sessions in order to eliminate the noise from the data [5]. But direct removal of these small sized sessions may result in loss of a significant amount of information especially when the number of small sessions is large. We propose a "Fuzzy Set Theoretic" approach to deal with this problem. Instead of directly removing all the small sessions below a specified threshold, we assign weights to all the sessions using a "Fuzzy Membership Function" based on the number of URLs accessed by the sessions.

Fig. 2 depicts a linear fuzzy membership function for session weight assignment. Here β_1 represents a lower threshold on the number of URLs accessed in a session and β_2 represents an upper threshold on the number of URLs accessed in a session.

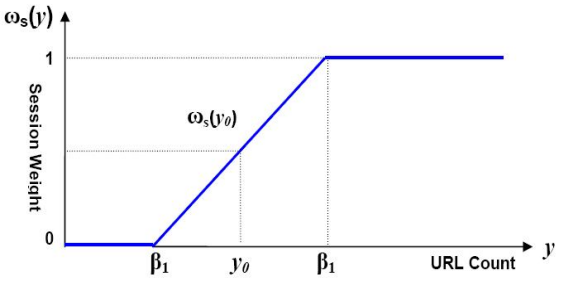

Fig. 2. Linear Fuzzy Membership Function for Session Weight Assignment

The session weight assignment using the linear fuzzy membership function takes is carried out using the following formula:

$$\omega_s(y) = \begin{cases} 0, \text{if } |s| \le \alpha_1 \\ 1, \text{if } |s| \ge \alpha_2 \\ \dfrac{|s|-\alpha_1}{\alpha_2-\alpha_1}, \text{otherwise} \end{cases} \quad (3)$$

where

ω_s is weight assigned to the sessions

β_1 is the lower threshold on the session URL count

β_2 is the upper threshold on the session URL count

$y = |s|$ is the number of URLs accessed in sessions

III. FUZZY C-MEANS CLUSTERING TECHNIQUE

Clustering techniques are widely used in WUM to capture similar interests and trends among users accessing a Web site. Details of various clustering techniques can be found in survey articles [13]-[15]. Fuzzy C-means clustering (FCM) incorporates the basic idea of Hard C-means clustering (HCM). In order to understand the Fuzzy C-means clustering let's first discuss the Hard c-means clustering algorithm, and then we will explore the details of Fuzzy c-means clustering algorithm.

A. Hard c-Means Clustering :

The Hard c-means clustering technique is [16] is a popular technique and has been applied to a variety of areas [17]-[19]. This algorithm relies on finding cluster centres by trying to minimize a cost function of dissimilarity measure. The Hard c-Means clustering algorithm [16] is one of the most commonly used methods for partitioning the data. Given a set of m data points $X = \{x_i \mid i = 1 \cdots m\}$, where each data point is a n-dimensional vector, k-means clustering algorithm aims to partition the m data points into k clusters ($k \leq m$) $C = \{c_1, c_2, \ldots, c_k\}$ so as to minimize an objective function J(V, X) of dissimilarity, which is the within-cluster sum of squares. In most cases the dissimilarity measure is chosen as the Euclidean distance. The objective function is an indicator of the distance of the n data points from their respective cluster centers. The objective function J, based on the distance between a data point x_i in cluster j and the corresponding cluster centre v_j, is defined in (4).

$$J(X,V) = \sum_{j=1}^{k} J_i(x_i, v_j) = \sum_{j=1}^{k} \left(\sum_{i=1}^{m} u_{ij} . d^2(x_i, v_j) \right), \qquad (4)$$

where, $J_i(x_i, v_j) = \sum_{i=1}^{m} u_{ij} . d^2(x_i, v_j)$,

is the objective function within cluster c_i ,

$u_{ij} = 1$, if $x_i \in c_j$ and 0 otherwise.

$d^2(x_i, v_j)$ is the disatnce between x_i and v_j

Euclidian distance between various data points and cluster centers can be calculated using (5).

$$d^2(x_i, v_j) = \left\| \sum_{k=1}^{n} x_k^{\,i} - v_k^j \right\|^2 \qquad (5)$$

where , n is the number of dimensions of each data point

x_k^i is the value of kth dimensions of x_i

v_k^j is the value of kth dimensions of v_j

The partitioned clusters are defined by a m□k binary membership matrix U, where the element u_{ij} is 1, if the ith data point x_i belongs to the cluster j, and 0 otherwise. Once the cluster centers $V = \{v_1, v_2, \ldots, v_k\}$, are fixed, the membership function u_{ij} that minimizes (4) can be derived as follows:

$$u_{ij} = \begin{cases} 1; \text{ if } d^2(x_i, v_j) \leq d^2(x_i, v_{j^*}) \; j \neq j^*, \forall \, j^* = 1, \cdots, k \\ 0; \text{otherwise} \end{cases} \qquad (6)$$

The equation (5) specifies that assign each data point x_i to the cluster c_j with the closest cluster center v_j. Once the membership matrix $U=[u_{ij}]$ is fixed, the optimal center v_j that minimizes (4) is the mean of all the data point vectors in cluster j :

Algorithm: Fuzzy *c*-Means clustering algorithm

Input: *k*, the number of clusters and Set of *m* data points $X=\{x_1, \ldots, x_m\}$.

Output: Set of *k* centroids, $V=\{v_1, \ldots, v_k\}$, corresponding to the clusters $C=\{c_1, \ldots, c_k\}$, and membership matrix $U=[u_{ij}]$.

Steps:

1) Initialize the membership matrix $U=[u_{ij}]$, by randomly selecting u_{ij} values between 0 and 1.

2) **repeat**

   i) Compute the cluster centers $V=\{v_1, \ldots, v_k\}$ using (12).

   ii) Update the membership matrix U using (13).

   iii) Compute the objective function $J(U,V,X)$ using (9). Stop if it below a certain threshold ε.

3) **until** $J_i - J_{i-1} < \varepsilon$ (where ε is error threshold and i is the current iteration)

$$v_j = \frac{1}{|c_j|} \sum_{i, x_i \in c_j}^{m} x_i \qquad (7)$$

where ,

$|c_j|$, is the size of cluster c_j and also $|c_j| = \sum_{i=1}^{m} u_{ij}$

B. Fuzzy c-Means Clustering:

Fuzzy c-means clustering was proposed by Dunn [20] and improved by Bezdek [21] and is frequently used in the field of data mining. In this technique each data point belongs to a cluster to a degree specified by a membership grade. As in hard c-means clustering, Fuzzy C-means clustering relies on minimizing a cost function of dissimilarity measure. Fuzzy C-means clustering (FCM) incorporates the basic idea of Hard C-means clustering (HCM), with the difference that in FCM each data point belongs to a cluster to a degree of membership grade, while in HCM every data point either belongs to a certain cluster or does not belong. Therefore FCM performs the fuzzy clustering in such a way that a given data point may belong to several clusters with the degree of belongingness specified by membership grades between 0 and 1. However, FCM still uses an objective function that is to be minimized while trying to partition the data set. The algorithm calculates the cluster centers and assigns a membership value to each data item corresponding to every cluster within a range of 0 to 1. The algorithm utilizes a fuzziness index parameter q where $q \in [1, \infty]$ [22] which determines the degree of fuzziness in the clusters. As the value of q reaches to 1, the algorithm works like a crisp partitioning algorithm. Increase in the value of q results in more overlapping of the clusters.

Let $X = \{x_i \mid i = 1 \cdots m\}$ be a set of n-dimensional data point vectors where m is the number of data points and each $x_i = \{x_1^i, x_2^i, \cdots, x_n^i\} \forall i = 1 \cdots m$. Let $V = \{x_j \mid j = 1 \cdots c\}$ represent a set of n-dimensional vectors corresponding to the cluster center corresponding to each of the c clusters and each

$v_j = \{v_1^j, v_2^j, \cdots, v_n^j\} \forall j = 1 \cdots c$ Let uij represent the grade of membership of data point xi in cluster j. $u_{ij} \in [1,0] \, \forall i = 1 \cdots m$ and $\forall j = 1 \cdots c$. The $n \times c$ matrix $U = [u_{ij}]$ is a fuzzy c-partition matrix, which describes the allocation of the data points to various clusters and satisfies the following conditions:

$$\left.\begin{aligned} \sum_{j=1}^{c} u_{ij} = 1, \forall i = 1 \cdots m \\ 0 < \sum_{j=1}^{c} u_{ij} < m, \forall j = 1 \cdots c \end{aligned}\right\} \quad (8)$$

The performance index J(U,V,X) of fuzzy c-mean clustering can be specified as the weighted sum of distances between the data points and the corresponding centers of the clusters. In general it takes on the form:

$$J(U,V,X) = \sum_{j=1}^{c} \sum_{i=1}^{m} u_{ij}^q d_{ij}^2 \left(\bar{x}_i, \bar{v}_j\right) \quad (9)$$

where ,

$q \in [1, \infty]$ is the fuzziness index of the clustering

$d_{ij}^2 \left(\bar{x}_i, \bar{v}_j\right)$ is the disatnce between $\bar{x}i$ and $\bar{v}j$

The distance $d_{ij}^2 \left(\bar{x}_i, \bar{v}_j\right)$ for weighted and non-weighted user sessions and URLs is calculated using equations (10) and (11) respectively:

$$d_{ij}^2 \left(\bar{x}_i, \bar{v}_j\right) = \sum_{k=1}^{n} w_s(x_i) \left\| w_u\left(x_k^i\right) \bar{x}_k^i - \bar{v}_k^j \right\| \quad (10)$$

$$d_{ij}^2 \left(\bar{x}_i, \bar{v}_j\right) = \sum_{k=1}^{n} \left\| w_u\left(x_k^i\right) \bar{x}_k^i - \bar{v}_k^j \right\| \quad (11)$$

$w_s(x_i)$ is the weight of the data point x_i and

$w_u\left(x_k^i\right)$ is the weight of kth feature of the data point x_i

Minimization of the performance Index J(U,V,X) is usually achieved by updating the grade of memberships of data points and centers of the clusters in an alternating fashion until convergence. This performance Index is based on the sum of the squares criterion. During each of the iterations, the cluster centers are updated as follows:

$$\bar{v}_j = \frac{\sum_{i=1}^{m} u_{ij}^q \bar{x}_i}{\sum_{i=1}^{m} u_{ij}^q} \quad (12)$$

Membership values are calculated by the following formula:

$$u_{ij} = \frac{\left(\frac{1}{d_{ij}^2\left(\bar{x}_i, \bar{v}_j\right)}\right)^{1/(q-1)}}{\sum_{k=1}^{n} \left(\frac{1}{d_{ij}^2\left(\bar{x}_i, \bar{v}_j\right)}\right)^{1/(q-1)}} \quad (13)$$

Fig. 3. Fuzzy c-Means Clustering Algorithm

Fig. 3 describes the algorithm for Fuzzy C-Means Clustering. In order to decide the number of optimum clusters for the data set X we use a validity function S which is the ratio of compactness to separation [22] as given below:

$$S = \frac{\sum_{j=1}^{c} \sum_{i=1}^{m} u_{ij}^2 \left\| \bar{x}_i - \bar{v}_j \right\|^2}{m. \min_{l \neq k} \left\| \bar{v}_l - \bar{v}_k \right\|^2} \quad (14)$$

for each $c = c_{\min}, \cdots, c_{\max}$

Let Ω_c denote the optimal candidate at each c then, the solution to the following minimization problem yields the most valid fuzzy clustering of the data set.

$$\min_{c_{\min} \le c \le c_{\max}} \left(\min_{\Omega_c} S \right) \quad (15)$$

IV. EXPERIMENTAL RESULTS

The Web access logs are taken from the P.A. College of Engineering, web site, at URL http://www.pace.edu.in. The site hosts a variety of information, including departments, faculty members, research areas, and course information. The Web access logs covered a period of one month, from February 1, 2011 to February 8, 2011. There were 12744 logged requests in total. In order to discover the clusters that exist in user access sessions of a web site, following steps are performed:

1. Pre-processed the web log data to discover the user sessions.
2. Performed the fuzzy feature evaluations and dimensionality reduction by assigning weights to the user sessions and URLs.
3. Applied the Fuzzy c-means clustering algorithm to the non-weighted user sessions and URLs obtained in step 1.
4. Applied the Fuzzy c-means clustering algorithm to the weighted user sessions and URLs obtained in step 2.
5. Compared the results of steps 4 and 5.
6. Details of the above experimental steps are described in the following sub-sections.

A. Web Usage Log Preprocessing

After performing the cleaning operation the output file contained 11995 entries. Total no. of unique users identified is 16 and the no. of user sessions discovered are 206. Table II depicts the results of cleaning and user session identification steps. Further details pre-processing can be found from our previous work [23].

Items	Count
Initial No of Log Entries	12744
Log Entries after Cleaning	11995
No. of site ULRs accessed	260
No of Users Identified	16
No. of User Sessions Identified	418

TABLE I
RESULTS OF Cleaning and user identification

B. Fuzzy Feature Evaluation and Dimensionality Reduction

Table I shows that the number of URLs appearing in the access log is 260. Since each user session is represented as a vector of URL items, we would like to reduce the dimensionality of user session vectors by evaluating the URL items and eliminating the most insignificant ones. We evaluate each URL item based on the session support count. For this purpose we choose the lower bound on the number of sessions supported by the URL (LB) as 1 and an upper bound on the number of sessions supported by the URL (UB) as 6. Using equation (2), weights assigned to various URL items are described in Table I.

URL Session Support	1	2	3	4	5	6+
URL Weight	0	0.2	0.4	0.6	0.8	1

TABLE I
URL Weights Based on The URL Access Count

Fig. 4 shows the result of URL evaluation. Note that 142 URLs have the session support count of 1, hence they are eliminated by assigning them weight 0. The elimination of 142 URL items is a big reduction in the dimensionality of each user session vector. Other less significant URLs are assigned weights as show in Table I.

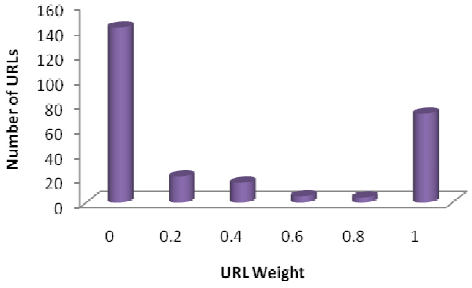

Fig. 4. Number of URLs vs. Associated URL Weight

C. Assigning Weights to User Sessions:

Table I shows that the number of user sessions discovered from the access log are 418. Since each user session is represented by a row in the data matrix, we would like to reduce the row dimensionality of the data matrix by evaluating the user sessions and eliminating the most insignificant ones. We evaluate each user session based on the number of URL items accessed in that session. For this purpose we choose the lower bound on the number of URLs accessed in a session (LB) as 1 and an upper bound on the number of URLs accessed in a session (UB) as 6. Using equation (3), weights assigned to various sessions are described in Table II.

Session URL Count	1	2	3	4	5	6+
Session Weight	0	0.2	0.4	0.6	0.8	1

TABLE II
Session Weights Based on The URL Access Count

Fig. 5 shows the result of user session weight assignment. Note that 212 use sessions access only a single URL, hence they are eliminated by assigning them a weight of 0. The elimination of 212 user sessions is a big reduction in the row dimensionality of the data matrix. Other less significant user sessions are assigned weights as show in Table II.

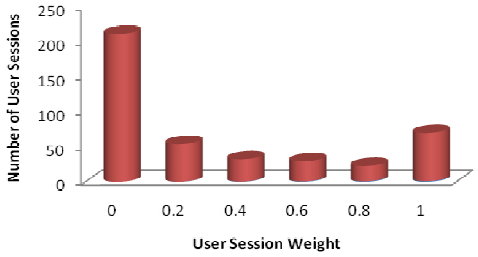

Fig. 5. Number of User Sessions vs. Associated Session Weight

The result of dimensionality reduction is show in the Table III below:

Items	Before Dimensionality Reduction	After Dimensionality Reduction
No. of URL items	260	118
No. of User Sessions	418	206

TABLE III
Results After Dimensionality Reduction

D. Mining User Session Clusters using Fuzzy c-Means Clustering :

Fuzzy c-Mean clustering algorithm (Fig. 3) is applied to discover session clusters that represent similar URL access patterns. The performance Index J(U,V,X) of Fuzzy c-mean clustering is calculated using equation (9). It is the weighted sum of distances between the data points and the corresponding centers of the clusters.

Fuzzy c-Mean clustering algorithms described in Fig. 3 is first applied by initializing k=2. During each of the iterations we increased the number of clusters by 1 till the number of clusters is reached to 67 (One third of total number of user sessions). We repeated the above process for weighted as well as non-weighted URLs and sessions. While applying the c-means clustering algorithm to weighted and non-weighted user sessions, the main difference lies in calculating the distance between data points and cluster centres. For weighted sessions and URLs these distances are computed using equation (10). On the other hand for non-weighted user sessions, distances are calculated using equation (11).

Graph is Fig. 6 shows the performance index (J) versus number of clusters for weighted as well as non-weighted URLs and sessions. From the graph it is clear that "Fuzzy Set Theoretic" weight assignment

to sessions and URLs results in better minimization of the performance index than non-weighted session approach.

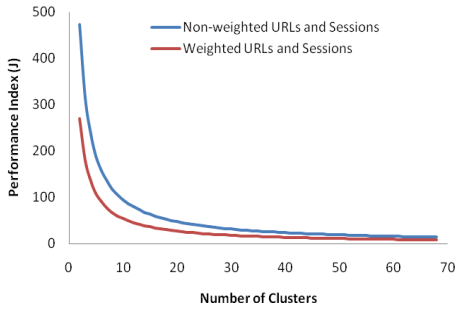

Fig. 6. Performance Index J vs. No. Of Clusters k

In order to decide the number of optimum clusters we calculated the validity index (S), which is the ratio of compactness to separation using the equation (14).

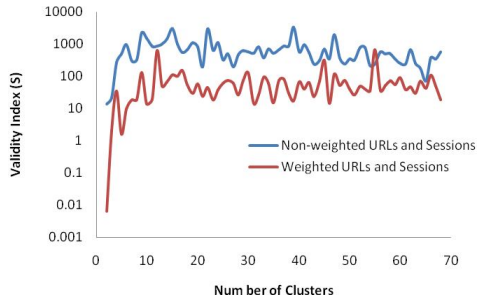

Fig. 7. Validity Index S vs. No. Of Clusters k

Fig. 7 provides the graphs of validity index (S) versus number of clusters for weighted and non-weighted URLs and user sessions respectively. Our results show that the fuzzy approach of URL and session weight assignment resulted in better minimization of clustering validity index than without session weight assignment. Our result also shows that the validity index is minimized when k=2, for weighted as well as non-weighted URLs and sessions.

CONCLUSION

In this paper, we discussed our methodology to perform feature evaluation and dimensionality reduction by assigning weights to URLs and user sessions using linear fuzzy membership functions. We also discussed the mathematical details about how to apply the Fuzzy c- Mean Clustering algorithm in order to cluster the user sessions.

In order improve the quality of the clustering; we proposed a "Fuzzy Set Theoretic" approach for feature evaluation and dimensionality reduction. Instead of directly removing all the low session support URLs below a specified threshold, we assign weights to the URLs using a fuzzy membership function based on the session support count of the URL. Similarly, instead of directly removing all the small sessions below a specified threshold, we assign weights to the sessions using another fuzzy Membership function based on the number of URLs accessed by the sessions. Finally we compared our soft computing based approach of URL and session weight assignment with the traditional hard computing based approach of low support URL and small session elimination. Our results show that the fuzzy set theoretic approach of URL and session weight assignment results in better minimization of clustering performance index than without session weight assignment. It also improves the validity index much better than without weight assignment.

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