STABILITY ANALYSIS FOR SOLUTIONS OF CU-WATER NANOFLUID FLOWS IN A CHANNEL WITH SLOWLY EXPANDING WALLS

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Abstract - This paper studies the stability of the triple solutions previously obtained by Raza et al. (2016) for the problem of flow and heat transfer of Cu-Water nanofluid in a channel with slowly expanding walls.Raza et al (2016) has successfully examined the occurrence of triple solutions for both cases of expanding and contracting walls. However, the stability analysis for the solutions obtained has not been considered. In order to perform the stability analysis, the unsteady case for the problem has been developed in this paper by introducing new dimensionless time variable. The equations are then solve numerically using bvp4c in Matlab. In this paper, the study is limited to the case of expanding walls and results show that only the first solution is stable while the remaining (second and third solution) are unstable. This study will complete the work of Raza et al. (2016) in terms on stability analysis for the multiple solutions occurred in the problem.

Keyword - Stability Analysis; Triple solutions; Nanofluids; Channel; Expanding Walls.

I. INTRODUCTION

The multiplicity of solutions in fluid dynamics and heat transfer is important to be able to be computed since solutions arising from bifurcations often interact with one another producing otherwise inexplicable phenomena [1]. The transition process provides valuable information of flow evolution and transition tomultiple solutions acts as a starting point to turbulence or chaos [2]. In heat transfer engineering, the flow multiplicity may significantly affect the structure and quality of the final product in material processing [3]. An improved understanding of the development of flow statesmultiplicity can stimulate innovations and lead to enhancement of performance andreliability as well as reducing the costs of various practical flow problems such asrotating machines and crystal growth processes [2].

Nowadays, it is widely understood that the computational modelling ofbifurcation as well as multiplicity of fluid flows are important, but the computationof all possible multiple solutions, remain a challenge [2]. In real situation, multiple solutions that may exist in boundary layer flow aredifficult to visualize. This can cause researchers fail to notice the multiple solutionsthat might exist within the flows, which is an important aspect of fluid mechanics [4]. Therefore, a mathematical computation and analysis are required todetermine the existence of multiple solutions.

The significance of computational modelling of bifurcation as well as multiplicity of fluid flows as highlighted above has lead Raza et al. [5] to study the occurrence of multiple solutions in the problem of Cu-Water nanofluid flows in a channel with slowly expanding and contracting walls. Raza et al [5] has successfully examined the occurrence of triple solutions for both cases of expanding and contracting walls. However, the stability analysis for the solutions obtained has not been considered in their previous work. Therefore, in this paper we proceed the work of Raza et al [5] to study the stability analysis on all the solutions obtained previously. This is done by formulating the general steady case of the problem. This study will serve as a complement to the work of Raza et al [5].

II. PROBLEM FORMULATION

Previously, Raza et al [5] considered a two-dimensional flow of unsteady, laminar and incompressible nanofluids in a porous channel where the channel walls are variant in the direction of y-axis and can be expanded or contracted with respect to the time dependent rate $\dot{a}$. Both channel walls are assumed to have the same permeability and uniform wall suction/injection is imposed at the walls. The fluid is considered symmetric about y-axis as shown in Fig. 1.

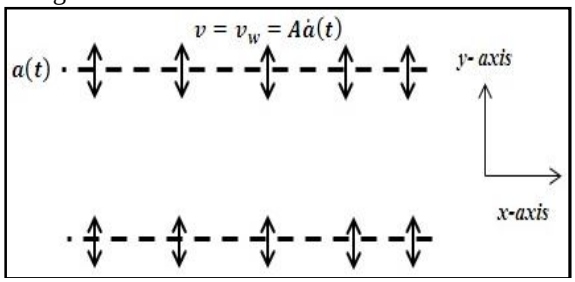

Fig. 1Physical model of the problem [5].

The boundary layer equations that represent for the problem as stated in [5] are as follows:

$$f'''' + A_1(1-\varphi)^{2.5}\left(\alpha[\eta f''' + 3f''] + R(ff''' - f'f'')\right) = 0 \quad (1)$$

$$\theta'' + \frac{A_1}{A_2}(PrfR + \alpha\eta Pr)\theta' = 0 \quad (2)$$

with boundary conditions:

$$\left.\begin{matrix} f(0) = 0, f''(0) = 0, \theta(0) = 1 \\ f(1) = 1, f'(1) = 0, \theta(1) = 0 \end{matrix}\right\} \quad (3)$$

III. STABILITY ANALYSIS

To perform the stability analysis for the above problem, we now formulate a general steady case for the problem by taking [6, 7],

$$f(\eta) = f_{\circ}(\eta) + e^{-\lambda t}F(\eta, t) \quad (4)$$

$$\theta(\eta) = \vartheta_{\circ}(\eta) + e^{-\lambda t}G(\eta, t) \quad (5)$$

where

$\tau = t$ and $0 < F(\eta, t) \ll 1$, $0 < G(\eta, t) \ll 1$ and λ is the unknown eigenvalues, $F(\eta, t)$ and $G(\eta, t)$ are smallest relative to $f_{\circ}(\eta)$ and $\theta_{\circ}(\eta)$ respectively.

Substituting Eq. 4 and Eq. 5 into Eq. 1 and Eq. 2, resulting the following equations:

$$\frac{\partial^4 f}{\partial \eta^4} + A_1(1-\varphi)^{2.5}\left[\alpha\left(\eta\frac{\partial^3 f}{\partial \eta^3} + 3\frac{\partial^2 f}{\partial \eta^2}\right) + R\left(f\frac{\partial^3 f}{\partial \eta^3} - \frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta^2}\right)\right] = \frac{\partial^3 f}{\partial \tau \partial \eta^2} \quad (6)$$

$$\frac{\partial^2 \theta}{\partial \eta^2} + Pr\frac{A_2}{A_3}(Rf + \alpha\eta)\frac{\partial \theta}{\partial \eta} = \frac{\partial \theta}{\partial \tau} \quad (7)$$

To analyse the stability of the steady state, we set $\tau = 0$ [6]. Then, Eq. 6 and Eq. 7 become:

$$F'''' + A_1(1-\varphi)^{2.5}\begin{bmatrix} \alpha\left(\eta f_{\circ}''' F'' + 3 f_{\circ}'' F'''\right) \\ + \\ R\left(f_{\circ} F''' + F f_{\circ}''' - f_{\circ}' F'' - f_{\circ}'' F'\right) \end{bmatrix} + \lambda F'' = 0 \quad (8)$$

$$\theta'' + \frac{A_2}{A_3}[PrRf_{\circ} + \alpha\eta Pr]\theta_{\circ}' + \frac{A_2}{A_3}[PrRF + \alpha\eta Pr]G'' + \lambda G = 0 \quad (9)$$

with the boundary conditions,

$$\left.\begin{matrix} F(1) = 1, F'(1) = 0, G(1) = 0 \\ F''(0) = 0, F(0), G(0) = 1 \end{matrix}\right\} \quad (10)$$

$f_{\circ}(\eta)$ and $\theta_{\circ}(\eta)$ can be determined by the smallest eigenvalue λ due to the steady state flow solution. Therefore, the range of the possible eigenvalues can be determined by relaxing the boundary conditions on $f_{\circ}(\eta)$ and $\theta_{\circ}(\eta)$ as prescribed by Harris et al. [8].

In this case, we relaxed the boundary condition $G(1) \to 0$ and solve the system of differential equation with the new boundary condition $G'(0) = 1$.

III. RESULTS AND DISCUSSION

Previously, Raza et. al [5] have solved Eq. 1 and Eq. 2 with the boundary conditions 3 by using shooting method. They have published the following results as shown in Fig. 2 and in Table 2. As we can see from Fig. 2, there occur triple solutions for the problem.

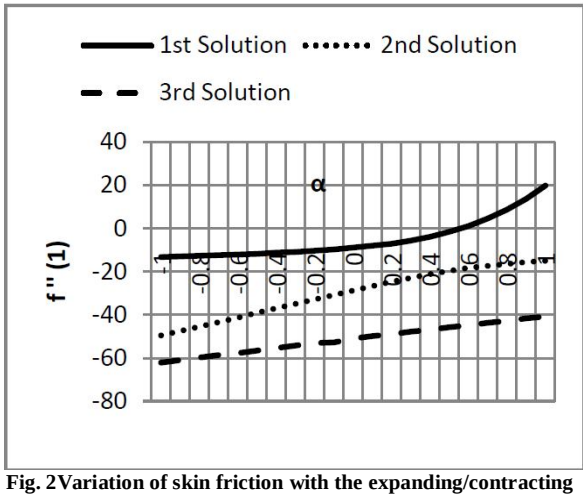

Fig. 2Variation of skin friction with the expanding/contracting parameter [5].

R	α	Pr	φ	$f''(-1)$
0	-0.1	6.2	0.1	-3.110833618
-10				-9.718933431
-20				-25.93558737
-10	-0.1	6.2	0.0	-6.369097305
			0.05	-8.248977654
			0.1	-9.718934067

Table 1 Numerical values for the solution of skin friction obtained by using shooting method [5].

α	1st Solution	2nd Solution	3rd Solution
	λ		
0	5.448	-0.5	-6.844
0.1	6.9873	-4.81	-8.8888
0.5	8.5266	-14.2438	-15.0805
1	10.0659	-19.479	-15.5839

Table 2 Smallest eigen values for triple solutions of skin friction when R = - 10, φ = 0.05 and Pr= 6.2.

Now, in this paper, we investigate the stability of solutions in Fig. 2 by solving Eq. 8 and Eq. 9 subject to boundary condition 10. To determine the stability

of solutions, we computed the smallest eigen values, λ for all solutions branches. Eq. 8 and Eq. 9 subject to boundary condition 10 have been solved by using bvp4c in Matlab Software. Table 2 shows the smallest eigen values, λobtained for each solution. From Table 2, we observed that only the first solution has positive value of λ while the remaining two solutions have negative value of λ. This denotes that only first solution is stable while second solution and the third solution are unstable. It is worth to highlight that, the eigenvalues shown in Table 2 are only for the expanding walls ($\alpha > 0$).

CONCLUSION

The stability analysis for the solutions obtained previously by [5] has been successfully performed in this paper by introducing new dimensionless time variable. The equations are then solve numerically using bvp4c in Matlab. We only consider the case of expanding walls in this paper and results show that only the first solution is stable while the remaining are unstable.

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