NEW TECHNOLOGY FOR THE SOLUTION OF PRIMAL (ALL-INTEGER) IN LINEAR PROGRAMMING

JAI RAM BHALLA

Applied Sciences Punjab Technical University, Punjab

Abstract- in this paper we provide a new technology for selection of pivot column and the source row in primal (all integer) linear programming, by using this technology, the number of iterations can be decreased by passing through the initial simplex table to the optimal simplex table. The source row automatically calculated by using this technology.

Key words- Pivot Column, The Source Row, Optimal Simplex Table, Initial Simplex Table.

I. INTRODUCTION

The computational procedure of simplex method requires to the construction of simplex table. The initial simplex table is constructed by writing out the coefficients & the constraints of linear programming in a systematic tabular form.

An iterative procedure for the solution of an all integer linear programming problem by Gomory's fractional cut method (or cutting plane method) can be summarised. All the current methods are based on the concept of selecting the pivot column & source row as given in linear programming for the solution of integer linear programming problem.

We have found the pivot column by minimum of the product of the negative coefficient of the objective function and the minimum of the ratios used in the source row. The source row automatically comes out by this technique. However the formulation of cut and the interation process of our technology are the same as the other well known methods.

II. THEORETICAL FOUNDATIONS

Consider the problem

$$\left.\begin{array}{ll} \text{Max} & x_0 = cx \\ \text{Such that} & Ax = b \\ & x \geq 0, \text{ integers} \end{array}\right\} \quad \ldots(i)$$

Which can be decreased to

$$x_{Bi} = y_{i0} - \sum_{j \varepsilon R} y_{ij}\, x_j \quad \ldots(ii)$$

Where $x_{B0} = x_0$, x_j's, $j \varepsilon R$ are non-basic variables, x_B j's are basic variables. A very detailed explanation of equ. (ii) is given in equ. (i). Basic variables, non-basic variables are shown in the initial simples table (except the bottom row)

Initial Simplex Table

Basic variable →		.....	$-x_j$	$-x_{k_t}$	$-x_{k_s}$ .....
Non-basic variables	x_0	y_{00}	y_{0j}	y_{0k_t}	y_{0k_s}
	:				
	:				
↓	x_{B_i}	y_{i0}	y_{ij}	y_{ikt}	y_{iks}
	:				
	:				
←	$x_{B_{r_t}}$	$y_{r_t 0}$	$y_{r_t j}$	$y_{r_t k_t}$	$y_{r_t k_s}$
Source row	:				
	:				
	$x_{B_{r_s}}$	$y_{r_s 0}$	$y_{r_s j}$	$y_{r_s k_t}$	$y_{r_s k_s}$
	:				
	:				
← cut	S	$\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]$	$\left[\frac{y_{r_t j}}{y_{r_t k_t}}\right]$	$\left[\frac{y_{r_t k_t}}{y_{r_t k_t}}\right]$	$\left[\frac{y_{r_t k_s}}{y_{r_t k_t}}\right]$

Assume that x_B is non-degenerate and $y_{oj} \leq -1$ for some $j \varepsilon R$ in (ii) say y_{ok_1}, y_{ok_2} ..... y_{ok_t} ..... y_{ok_s} ..... y_{ok_p}. Then by increasing any one of x_{k_1}, x_{k_2} ..... x_{k_1} ..... x_{k_s} ..... x_{k_p} and keeping all other non-basic variables fixed at zero, x_{B_0} increases linearly with slopes. $-y_{ok_1}$, $-y_{ok_2}$

$\ldots\ldots -y_{ok_3} \ldots\ldots -y_{ok_4} \ldots\ldots -y_{ok_n}$ respectively.

Because x is required to be integer,

$$x_{k_t} + \sum_{j \in \mathrm{R}-\{k_t\}} \left[\frac{y_{r_t j}}{y_{r_t k_t}}\right] x_j + 0 \le \left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]$$

i.e., s =

$$\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right] - \sum_{j \in \mathrm{R}-\{k_t\}} \left[\frac{y_{r_t j}}{y_{r_t k_t}}\right] - x_{k_t} \quad \ldots.(v)$$

Here s is the slack variable Here (iv) is the addition constraint called cut which is given at the bottom of the initial simplex be :

Eliminating x_{k_t} from (ii) by using (iv) to get

$$x_{\mathrm{B}_i} = \left(y_{io} - y_{ik_t}\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]\right) - \sum_{j \varepsilon \mathrm{R}-\{k_t\}} \left(y_{ij} - y_{ik_t}\left[\frac{y_{r_t j}}{y_{r_t k_t}}\right]\right) - \left(-y_{ik_t}\right) s$$

From (iv) and (v) the transformed simplex table given below :

Transformed Simplex Table

	$\ldots\ldots\ldots\ldots$	$-x_j \ldots\ldots\ldots$	$-\mathrm{S} \ldots\ldots$
x_0 : :	$y_{00} - y_{0k_t}\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]$	$y_{0j} - y_{0k_t}\left[\frac{y_{r_t j}}{y_{r_t k_t}}\right]$	y_{0k_t}
x_{B_i} : :	$y_{i0} - y_{ik_t}\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]$	$y_{ij} - y_{ik_t}\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]$	$-y_{ik_t}$
$x_{\mathrm{B}_{r_t}}$: :	$y_{r_t 0} - y_{r_t k_t}\left[\frac{y_{r_t 0}}{y_{r_t k_t}}\right]$	$y_{r_t j} - y_{r_t k_t}\left[\frac{y_{r_t j}}{y_{r_t k_t}}\right]$	$-y_{r_t k_t}$
x_{k_t}	$\left[y_{r_t 0}\right]$	$\left[\frac{y_{r_t i}}{y}\right]$	1

III. ALGORITHM FOR THE SOLUTION OF LPP PROBLEM

Step 1. From the initial simples table according to (ii)

Step 2. Test whether all $y_{0j} \ge 0$. If yes, the present solution is optimal. If not go to step (3).

Step 3. Calculate $\phi_{r_j k_j}$ for all j such that $y_{0j} \le -1$ by

$$\phi_{r_j k_j} = \min_{i=1,2....m} \left\{\left[\frac{y_{i0}}{y_{ik_j}}\right],\ y_{ik_j} \ge 1\right\}$$

Step 4. Now Select r_t and k_t by

$$\phi_{r_t k_t} = \min_{j=1,2....p} \left\{\phi_{r_j k_j}\ y_{ok_j}\right\}$$

Step 5. For the source row x_{r_t} and pivot column x_{k_t} construct the cut (4).

Step 6. Complete the transformed simplex table by the transformation (4) and (5).

Step 7. Go to step (2) for optimality test.

CONCLUSION

It is more rapid than most other known methods. This technique may be useful in dual all-integer programming mixed integer linear programming and bounded value integer linear programming.

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