SUM NORM APPROACH BASED DECISION MAKING

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Abstract - In this paper, two methods of decision making have been developed. An algorithm of membership functions and a fuzzy soft set have been proposed in order to get the optimal alternative solution for decision making in which all alternatives depend on a same set of parameters. The first method, sum norm and adjustable sum norm method based on suitable values is used to decide a sample matching an optimal alternative. The process by calculating sum norm and adjustable sum norm will be used. The second method is a sum norm and adjustable sum norm method based on fuzzy sets which is more efficient than the first one because in the process of this method, it exploits any membership functions of fuzzy sets with respect to each pair of parameters and alternatives. Moreover, the particular membership functions will be constructed in order to be shown that the optimal alternative solution from two methods are the same. Finally, both methods are applied to some problem involving the making decision.

Index Terms - Decision making, membership function, fuzzy soft set, sum norm, adjustable sum norm.

I. INTRODUCTION

The real world problems are uncertain. So, mathematical notions have to be exactly, otherwise certain logic would not be possible. In recent years, fuzzy knowledge has used in many practical problems such as economics, engineering, environmental science, and medical science. When approximate reasoning is done, decision making in a fuzzy soft sets can be proposed to classify a lot of information that helps us to find the optimal alternative for a sample.

The previous work of Tanushree Mitra Basu [7] studied decision making in a fuzzy soft set and his approach is based on Feng's method [2]. Feng made his decision making by a top level. In case that it was still not worked, Feng used mid-level to be α-cut to make the decision again. Nevertheless, Feng's method still had a problem that some making decision problems, the optimal solution was not unique. So the next step of Tanushree Mitra Basu was to study the decision making by the Analytical Hierarchy Process [6].

In Analytical Hierarchy Process, there was still a problem similar like in Feng's method that some decision making problem, it was still impossible to find unique optimal alternative. Third step of Tanushree Mitra Basu's research was to study about Naive Bayes' classification method [4]. This method used statistics in order to classify data. Nevertheless, when Bayes' method was used for certain data like disease symptoms, it was still unable to find the optimal alternative as a unique disease. Based on these previous methods he finally created a new method in order to find the optimal alternative which is unique by using mean potential approach. So that it is the α-cut for his making decision. This method works also for many decision making problems when Feng's method, analytical hierarchy process, and Naive Bayes' classification method have failed. In this paper improved methods for fuzzy soft set decision making process are presented to match a sample to an optimal alternative. The methods consist of sum norm and adjustable sum norm method based on suitable values, and sum norm and adjustable sum norm method based on fuzzy sets.

The outline of the paper is as following. In section 2, basic definitions and required results in this research are summarized. In section 3, algorithms for finding the optimal alternative are given in two exact processes. In section 4, two theorems, one lemma, and one corollary show that the same optimal solution is arrived when the particular membership function are constructed. In section 5 and section 6, the application of the results to some problems are described and the conclusion are made.

II. PRELIMINARIES

Before parlance of decision making, definitions about soft set, suitable soft set, and fuzzy soft set will be mentioned as follow:

Throughout this paper let $U = \{u_1, u_2, ..., u_m\}$ be an initial universe set and $E = \{x_1, x_2, ..., x_n\}$ be a set of parameter, 2^U denotes the power set of U, and $A \subseteq E$. For definitions 1, 3, and 4 see [1], [3], and [5].

Definition 1 Let a mapping given by $f_A : E \to 2^U$ such that $f_A(x) = V_x$ where $x \in A$ for some $V_x \in U$ and $f_A(x) = \varnothing$ where $x \in A^c$ respectively.

A pair (E, f_A) is called a soft set over U. Thus a soft set (E, f_A) over U can be represented by the set of ordered pairs i.e., $f_A = \{(x, V_x) : x \in A, V_x \in 2^U\} \cup \{(x, \varnothing) : x \in E - A\}$

Definition 2 Let $\Gamma = \{f : U \to \mathbb{R}\}$ and $\psi_A : A \to \Gamma$. For each $x \in A, \psi_A(x)$ is denoted by $\psi_A^{(x)}$. The value $\psi_A(x)$ is called suitable value for U with respect to a parameter x. The function ψ_A is called a suitable value function for U with respect to the parameter set A.

Then ψ_A can be represented by the following Table 1.

Γ_A	x_1	x_2	$\cdots$	x_n
u_1	$\psi_A^{(x_1)}(u_1)$	$\psi_A^{(x_2)}(u_1)$	$\cdots$	$\psi_A^{(x_n)}(u_1)$
u_2	$\psi_A^{(x_1)}(u_2)$	$\psi_A^{(x_2)}(u_2)$	$\cdots$	$\psi_A^{(x_n)}(u_2)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\psi_A^{(x_1)}(u_m)$	$\psi_A^{(x_2)}(u_m)$	$\cdots$	$\psi_A^{(x_n)}(u_m)$

Table 1: The table represents suitable value from a model

If $\psi_A^{(x_j)}(u_k) = a_{kj}$ for all $k=\{1,2,\ldots,m\}$ and $j=\{1,2,\ldots,n\}$ then the suitable value function for U with respect to A is uniquely characterized by a matrix

$$M_{\psi_A} = [a_{kj}]_{m\times n} = \begin{pmatrix} a_{11} & a_{11} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

Then the matrix M_{ψ_A} is called suitable value matrix of ψ_A over U.

Definition 3 Let a fuzzy set A over U is a set defined be a function μ_A representing a mapping $\mu_A : U \to [0,1]$, μ_A is called the membership function of A, and the value $\mu_A(u)$ is call the grade of membership of u. Thus, a fuzzy set A over U can be represented as follows:

$$A = \{(u, \mu_A(u)) : u \in U\}$$

Note that the set of all the fuzzy sets over U will be denoted by $F(U)$.

Definition 4 Let $F(U)$ denotes the set of all the fuzzy sets over U and a pair (E, f_A) be a soft set over U. The mapping is given by $\gamma_A : E \to F(A)$ such that $\gamma_A(x) = f_A(x)$ for all $x \subset E$. That is

$$\gamma_A(x) = \begin{cases} V_x \in F(U) & ; x \in E \\ \varnothing & ; x \notin A \end{cases}$$

A pair (E, γ_A) is called is called a fuzzy soft set generated from a soft set (E, f_A) over U. Thus a fuzzy soft set generated from a soft set (E, γ_A) over U can be represented by the set of ordered pairs

$$\gamma_A = \{(x, V_x) : x \in E, V_x \in F(U)\}.$$

Definition 5 Let ψ_A be a suitable value function of U with respect to the parameter set A and M_{ψ_A} be a suitable value matrix over U. For each $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in \mathbb{R}^n$, define an absolute different matrix of M_{ψ_A} and α by

$$[|M_{\psi_A} - \alpha|] = \begin{pmatrix} |a_{11} - \alpha_1| & |a_{12} - \alpha_2| & \cdots & |a_{1n} - \alpha_n| \\ |a_{21} - \alpha_2| & |a_{22} - \alpha_2| & \cdots & |a_{2n} - \alpha_{2n}| \\ \vdots & \vdots & \ddots & \vdots \\ |a_{m1} - \alpha_1| & |a_{m2} - \alpha_2| & \cdots & |a_{mn} - \alpha_n| \end{pmatrix}$$

For each positive integer k such that $1 \le k \le m$, define

$$z_k = (|a_{k1} - \alpha_1|, |a_{k2} - \alpha_2|, \ldots, |a_{kn} - \alpha_n|)$$

Then the matrix $[|M_{\psi_A} - \alpha|]$ can be written in the form $[z_k]_{k=1}^m$.

Definition 6 Let ψ_A be a suitable value function of U with respect to the parameter set A and M_{ψ_A} be a suitable value matrix over U. For each $1 \le k \le m$, and $1 \le j \le n$, let $\tilde{B}_{kj}$ be a fuzzy set with respect the parameter α_j and the alternative u_k on $[b_j, c_j]$ with the membership function $\mu_{\tilde{B}_{kj}}$ in which $\mu_{\tilde{B}_{kj}}(a_{kj}) = 1$.

1. Then clearly a fuzzy matrix $M_{\psi_A}^{\tilde{B}}$ is defined by $M_{\psi_A}^{\tilde{B}} = [\mu_{\tilde{B}_{kj}}(a_{kj})]_{m\times n}$. Clearly, all its elements are 1's.

2. For each $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)\in\mathbb{R}^n$, defined a matrix $M_\alpha^{\tilde{B}}$ by $M_\alpha^{\tilde{B}}=[\mu_{\tilde{B}_{ij}}(\alpha_j)]_{m\times n}$.

3. For each $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)\in\mathbb{R}^n$, defined an absolute matrix $M_{\psi_A}^{\tilde{B}}$ and $M_\alpha^{\tilde{B}}$,

$$[|M_{\psi_A}^{\tilde{B}}-M_\alpha^{\tilde{B}}|]=\begin{pmatrix}|1-\mu_{\tilde{B}_{11}}(\alpha_1)| & |1-\mu_{\tilde{B}_{12}}(\alpha_2)| & \cdots & |1-\mu_{\tilde{B}_{1n}}(\alpha_n)|\\ |1-\mu_{\tilde{B}_{21}}(\alpha_1)| & |1-\mu_{\tilde{B}_{22}}(\alpha_2)| & \cdots & |1-\mu_{\tilde{B}_{2n}}(\alpha_n)|\\ \vdots & \vdots & \ddots & \vdots\\ |1-\mu_{\tilde{B}_{m1}}(\alpha_1)| & |1-\mu_{\tilde{B}_{m2}}(\alpha_2)| & \cdots & |1-\mu_{\tilde{B}_{mn}}(\alpha_n)|\end{pmatrix}$$

For each positive integer k such that $1\le k\le m$, define

$$z_k^{\tilde{B}}=(|1-\mu_{\tilde{B}_{k1}}(\alpha_1)|,|1-\mu_{\tilde{B}_{k2}}(\alpha_2)|,...,|1-\mu_{\tilde{B}_{kn}}(\alpha_n)|)$$

Then the matrix $[|M_{\psi_A}^{\tilde{B}}-M_\alpha^{\tilde{B}}|]$ can be written in the form $[z_k^{\tilde{B}}]_{k=1}^m$.

Definition 7 Let $y=(y_1,y_2,...,y_n)\in\mathbb{R}^n$.

1. A sum norm $\|.\|_s$ on $\mathbb{R}^n$ is defined by

$$\|y\|_s=\sum_{k=1}^n|y_k| \text{ for all } y=(y_1,y_2,...,y_n)\in\mathbb{R}^n.$$

2. An adjustable sum norm $\|.\|_p$ on $\mathbb{R}^n$ is defined by

$$\|y\|_p=\sum_{k=1}^n|y_k|+\sum_{j=1}^n\sum_{k>j}^n\|y_j|-|y_k\| \text{ for all } y=(y_1,y_2,...,y_n)\in\mathbb{R}^n.$$

It is clearly to prove that $\|.\|_s$ and $\|.\|_p$ are norms on $\mathbb{R}^n$.

Remark 8 Let ψ_A be a suitable value function of U with respect to the parameter set A and M_{ψ_A} be a suitable value matrix over U. For each $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)\in\mathbb{R}^n$, $1\le k\le m$ and $1\le j\le n$, let $\tilde{B}_{kj}$ be a fuzzy set on $[b_j,c_j]$ and $\alpha_j\in[b_j,c_j]$ with the membership function $\mu_{\tilde{B}_{kj}}(\alpha_j)$.

For the matrices $[|M_{\psi_A}-\alpha|]=[z_k]_{k=1}^m$ and $[|M_{\psi_A}^{\tilde{B}}-M_\alpha^{\tilde{B}}|]=[z_k^{\tilde{B}}]_{k=1}^m$. We obtain a sum norm base on suitable values $\|z_k\|_s$, a sum norm based on fuzzy sets $\|z_k^{\tilde{B}}\|_s$, an adjustable sum norm base on suitable values $\|z_k\|_p$, and an adjustable sum norm based on fuzzy sets $\|z_k^{\tilde{B}}\|_p$ as follows:

$$\|z_k\|_s=\sum_{k=1}^n|a_{kj}-\alpha_j|,$$

$$\|z_k^{\tilde{B}}\|_s=\sum_{k=1}^n|1-\mu_{\tilde{B}_{kj}}(\alpha_j)|,$$

$$\|z_k\|_p=\sum_{k=1}^n|a_{kj}-\alpha_j|+\sum_{k=1}^n\sum_{l>j}^n\|a_{kj}-\alpha_j|-|a_{kl}-\alpha_l\|,$$

$$\|z_k^{\tilde{B}}\|_s=\sum_{k=1}^n|1-\mu_{\tilde{B}_{kj}}(\alpha_j)|+\sum_{k=1}^n\sum_{l>j}^n\|1-\mu_{\tilde{B}_{kj}}(\alpha_j)|-|1-\mu_{\tilde{B}_{kl}}(\alpha_l)\|.$$

III. ALGORITHMS FOR FINDING THE OPTIMAL SOLUTION

In many decision making problems, there is a selection of optimal alternative from a list of alternatives. Improved methods are created to select an optimal alternative to match with a sample. The methods consist of sum norm and adjustable sum norm method based on suitable values, and sum norm and adjustable sum norm method based on fuzzy sets.

A. Sum norm and adjustable sum norm method based on suitable values

Let $M_{\psi_A}=[a_{ij}]_{m\times n}$ be a suitable value matrix over U. For each $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)\in\mathbb{R}^n$. For the matrix $[|M_{\psi_A}-\alpha|]=[z_k]_{k=1}^m$,

$$\text{Set } \sigma=\min_{1\le k\le m}\|z_k\|_s$$

Case 1 If there exists just only positive integer k^* with $1\le k^*\le m$ such that $\|z_{k^*}\|_s=\sigma$ i.e., $\|z_{k^*}\|_s<\|z_k\|_s$ for all $k\in\{1,2,...,m\}-\{k^*\}$, then the solution of the decision making problem is u_{k^*}. It is implies the suitable alternative u_{k^*} will be selected as optimal unique decision for the sample X.

Case 2 If there exists just only positive integer $k_1,k_2,...,k_q\in\{1,2,...,m\}$ with $1\le q\le m$, such that

$$\|z_{k_1}\|_s=\|z_{k_2}\|_s=\cdots=\|z_{k_q}\|_s=\sigma$$

then we will use the adjustable sum norm to help with solving the equation of the decision making problem. For each $i\in\{1,2,...,q\}$, the values of norms $\|z_{k_1}\|_p,\|z_{k_2}\|_p,...,\|z_{k_q}\|_p$ are computed, such that

$$\|z_{k_i}\|_p=\sum_{j=1}^n|a_{k_ij}-\alpha_j|+\sum_{j=1}^n\sum_{l>j}^n\|a_{k_ij}-\alpha_j|-|a_{k_il}-\alpha_l\|$$

Next, we consider $\gamma=\min_{1\le i\le q}\|z_{k_i}\|_p$

Case 2.1 If there exists just only positive integer k^* in $\{k_1,k_2,...,k_q\}$ such that

$$\|z_{k^*}\|_p-\gamma=\min_{1\le i\le q}\|z_{k_i}\|_p$$

i.e., $\|z_{k^*}\|_p\le\|z_{k_i}\|_p$ for all $i\in\{1,2,...,q\}$, then the alternative choice u_{k^*} will be assigned for the example X.

Case 2.2 If $\{k_1^*,k_2^*,...,k_t^*\}$ is the largest subset of $\{k_i:1,2,...,q\}$ in which

$$\|z_{k_1^*}\|_p=\|z_{k_2^*}\|_p=\cdots=\|z_{k_t^*}\|_p=\gamma,$$

Then the alternates $u_{k_1^*},u_{k_2^*},...,u_{k_t^*}$ are distributed for the sample X.

B. Sum norm and adjustable sum norm method based on fuzzy sets

Let $M_{\psi_A}=[a_{kj}]_{m\times n}$ be a suitable value matrix over U. For each $\alpha=(\alpha_1,\alpha_2,...,\alpha_n)\in\mathbb{R}^n$. For the matrix $[|M_{\psi_A}^{\tilde{B}}-M_\alpha^{\tilde{B}}|]=[z_k^{\tilde{B}}]_{k=1}^m$,

Set $\sigma = \min_{1\le k\le m} \| z_k^B \|_s$

Case 1 If there exists just only positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*}^B \|_s = \sigma$ i.e., $\| z_{k^*}^B \|_s < \| z_k^B \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$, then the solution of the decision making problem is u_{k^*}. It is implies the suitable alternative u_{k^*} will be selected as optimal unique decision for the sample X.

Case 2 If there exists just only positive integer $k_1, k_2, ..., k_q \in \{1,2,...,m\}$ with $1 \le q \le m$, such that

$$\| z_{k_1}^B \|_s = \| z_{k_2}^B \|_s = \cdots = \| z_{k_q}^B \|_s = \sigma$$

then we will use the adjustable sum norm to help with solving the equation of the decision making problem. For each $i \in \{1,2,...,q\}$, the values of norms $\| z_{k_1}^B \|_p, \| z_{k_2}^B \|_p$,...,$\| z_{k_q}^B \|_p$ are computed, such that

$$\| z_{k_i}^B \|_p = \sum_{j=1}^{n} |1 - \mu_{B_{kj}}(\alpha_j)| + \sum_{j=1}^{n}\sum_{l>j}^{n} \| 1 - \mu_{B_{kj}}(\alpha_j)| - |1 - \mu_{B_{kj}}(\alpha_l)\|$$

Next, we consider $\gamma = \min_{1\le i\le q} \| z_{k_i}^B \|_p$

Case 2.1 If there exists just only positive integer k^* in $\{k_1, k_2, ..., k_q\}$ such that

$$\| z_{k^*}^B \|_p = \gamma = \min_{1\le i\le q} \| z_{k_i}^B \|_p$$

i.e., $\| z_{k^*} \|_p \le \| z_{k_i} \|_p$ for all $i \in \{1,2,...,q\}$, then the alternative choice u_{k^*} will be assigned for the example X.

Case 2.2 If $\{k_1^*, k_2^*, ..., k_t^*\}$ is the largest subset of $\{k_i : 1,2,...,q\}$ in which

$$\| z_{k_1^*}^B \|_p = \| z_{k_2^*}^B \|_p = \cdots = \| z_{k_q^*}^B \|_p = \gamma,$$

Then the alternates $u_{k_1^*}, u_{k_2^*}, ..., u_{k_t^*}$ are distributed for the sample X.

IV. Main Results

In this section, lemma, theorems, and corollary are presented. They are important tools to prove that method III-A and method III-B are hold and related with each other. We begin with the particular membership function which is introduced in lemma 9. Then, theorem 10 and 11 are utilized in order to prove in corollary 12. Consequently, corollary 12 is shown the equivalent optimal solution that is obtained by method III-A and III-B under the particular membership function.

Lemma 9 Let $\psi_A = [a_{ij}]_{m\times n}$ be a suitable value matrix over U. For each $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n) \in \mathbb{R}^n$ such that $\lceil M_{\psi_A} - \alpha \rceil = [z_k]_{k=1}^m$. Let $1 \le k \le m$, and $1 \le j \le n$, there is a particular membership function on $[b_j, c_j]$ which maps a_{kj} to 1.

Set $d = \max_{1\le j\le n} |c_j - b_j|$

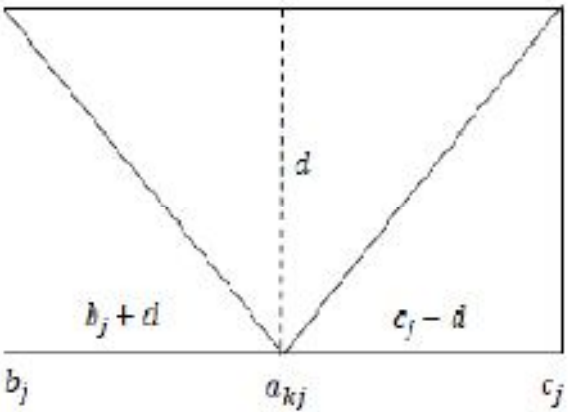


Fig.1. Construction of particular membership function.

Define a function f_{kj} on $[b_j, c_j]$ by $f_{kj}(\alpha_j) = |a_{kj} - \alpha_j|$ for all $\alpha_j, a_{kj} \in [b_j, c_j]$. Then we can transform the range of f_{kj} by defining the new function to the interval [0,1];

$$\mu_{kj}(\alpha_j) = 1 - \frac{f_{kj}(\alpha_j)}{d} \text{ for all } \alpha_j \in [b_j, c_j].$$

Clearly, $\{(\alpha_j, \mu_{kj}(\alpha_j)) | \alpha_j \in [b_j, c_j]\}$ is a fuzzy set on $[b_j, c_j]$ which is denoted by B_{kj} and also μ_{kj} is denoted by $\mu_{B_{kj}}$.

Theorem 10 Let $\psi_A = [a_{kj}]_{m\times n}$ be a suitable value matrix over U. For each $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n) \in \mathbb{R}^n, 1 \le k \le m$ and $1 \le j \le n$, let B_{kj} be a fuzzy set on $[b_j, c_j]$ with the membership function $\mu_{B_{kj}}$ in which $\mu_{B_{kj}}(a_{kj}) = 1$ and $\mu_{kj}(\alpha_j) = 1 - \frac{|a_{kj} - \alpha_j|}{d}$ for all $\alpha_j \in [b_j, c_j]$, where $d = \max_{1\le j\le n} |c_j - b_j|$. The following statements are equivalent:

1. If there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*} \|_s = \min_{1\le k\le m} \| z_k \|_s$ and $\| z_{k^*} \|_s < \| z_k \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$, then $\| z_{k^*}^B \|_s = \min_{1\le k\le m} \| z_k^B \|_s$ and $\| z_{k^*}^B \|_s < \| z_k^B \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$.
2. If there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*}^B \|_s = \min_{1\le k\le m} \| z_k^B \|_s$ and $\| z_{k^*}^B \|_s < \| z_k^B \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$, then $\| z_{k^*} \|_s = \min_{1\le k\le m} \| z_k \|_s$ and $\| z_{k^*} \|_s < \| z_k \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$.

Proof: "$\rightarrow$" Assume that there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*} \|_s = \min_{1\le k\le m} \| z_k \|_s$ and $\| z_{k^*} \|_s < \| z_k \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$. Consider

$$\| z_{k^*}^B \|_s = \sum_{j=1}^{n} |1 - \mu_{B_{k^*j}}(\alpha_j)|$$

$$= \sum_{j=1}^{n} \frac{|a_{k^*j} - \alpha_j|}{d}$$

$$= \frac{1}{d} \min_{1\le k\le m} \left(\sum_{j=1}^{n} |a_{kj} - \alpha_j|\right) \qquad \text{(by the assumption)}$$

$$= \min_{1\le k\le m} \left(\sum_{j=1}^{n} |1 - (1 - \frac{|a_{kj} - \alpha_j|}{d})\right)$$

$$= \min_{1\le k\le m} \left(\sum_{j=1}^{n} |1 - \mu_{B_{kj}}(\alpha_j)|\right)$$

$$= \min_{1\le k\le m} \| z_k^B \|_s . \qquad \square$$

And it is clearly to obtain that $\| z_{k^*}^B \|_s < \| z_k^B \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$ also.

"$\leftarrow$" Assume that there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*}^B \|_s = \min_{1\le k\le m} \| z_k^B \|_s$ and $\| z_{k^*}^B \|_s < \| z_k^B \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$. Consider

$$\| z_{k^*} \|_s = \sum_{j=1}^{n} |a_{k^*j} - \alpha_j|$$

$$= d \sum_{j=1}^{n} |1 - (1 - \frac{|a_{k^*j} - \alpha_j|}{d})|$$

$$= d \sum_{j=1}^{n} |1 - \mu_{B_{k^*j}}(\alpha_j)|$$

$$= d \min_{1\le k\le m} \sum_{j=1}^{n} |1 - \mu_{B_{kj}}(\alpha_j)| \qquad \text{(by the assumption)}$$

$$= \min_{1\le k\le m} \sum_{j=1}^{n} |a_{kj} - \alpha_j|$$

$$= \min_{1\le k\le m} \| z_k \|_s . \qquad \square$$

Moreover, it is clearly to obtain that $\| z_{k^*} \|_s < \| z_k \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$ also.

Theorem 11 Let $\psi_A = [a_{kj}]_{m\times n}$ be a suitable value matrix over U. For each $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n) \in \mathbb{R}^n, 1 \le k \le m$ and $1 \le j \le n$, let B_{kj} be a fuzzy set on $[b_j, c_j]$ with the membership function $\mu_{B_{kj}}$ in which $\mu_{B_{kj}}(a_{kj}) = 1$ and $\mu_{kj}(\alpha_j) = 1 - \frac{|a_{kj} - \alpha_j|}{d}$ for all $\alpha_j \in [b_j, c_j]$, where $d = \max_{1\le j\le n} |c_j - b_j|$. The following statements are equivalent:

1. If there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*} \|_p = \min_{1\le k\le m} \| z_k \|_p$ where $\| z_k \|_p = \| z_k \|_s + \sum_{j=1}^{n} \sum_{l>j}^{n} | |a_{kj} - \alpha_j| - |a_{kl} - \alpha_l| |$ and $\| z_{k^*} \|_p < \| z_k \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$, then $\| z_{k^*}^B \|_p = \min_{1\le k\le m} \| z_k^B \|_p$ and $\| z_{k^*}^B \|_p < \| z_k^B \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$.
2. If there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*}^B \|_p = \min_{1\le k\le m} \| z_k^B \|_p$ and $\| z_{k^*}^B \|_p < \| z_k^B \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$, then $\| z_{k^*} \|_p = \min_{1\le k\le m} \| z_k \|_p$ and $\| z_{k^*} \|_p < \| z_k \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$.

Proof: "$\rightarrow$" Assume that there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*} \|_p = \min_{1\le k\le m} \| z_k \|_p$ and $\| z_{k^*} \|_s < \| z_k \|_s$ for all $k \in \{1,2,...,m\} - \{k^*\}$. Consider $\| z_{k^*}^B \|_p$

$$= \sum_{k=1}^{n} |1 - \mu_{B_{k^*j}}(\alpha_j)| + \sum_{k=1}^{n} \sum_{l>j}^{n} | |1 - \mu_{B_{k^*j}}(\alpha_j)| - |1 - \mu_{B_{k^*l}}(\alpha_l)| |$$

$$= \sum_{k=1}^{n} \frac{|a_{k^*j} - \alpha_j|}{d} + \sum_{k=1}^{n} \sum_{l>j}^{n} | \frac{|a_{k^*j} - \alpha_j|}{d} - \frac{|a_{k^*l} - \alpha_l|}{d} |$$

$$= \frac{1}{d} \min_{1\le k\le m} \left(\sum_{k=1}^{n} |a_{kj} - \alpha_j| + \sum_{k=1}^{n} \sum_{l>j}^{n} | |a_{kj} - \alpha_j| - |a_{kl} - \alpha_l| |\right)$$

$$= \min_{1\le k\le m} \left(\sum_{k=1}^{n} \frac{|a_{kj} - \alpha_j|}{d} + \sum_{k=1}^{n} \sum_{l>j}^{n} | \frac{|a_{kj} - \alpha_j|}{d} - \frac{|a_{kl} - \alpha_l|}{d} |\right)$$

$$= \min_{1\le k\le m} \| z_k^B \|_p . \qquad \square$$

And it is clearly to obtain that $\| z_{k^*}^B \|_p < \| z_k^B \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$ also.

"$\leftarrow$" Assume that there exists just only one positive integer k^* with $1 \le k^* \le m$ such that $\| z_{k^*}^B \|_p = \min_{1\le k\le m} \| z_k^B \|_p$ and $\| z_{k^*}^B \|_p < \| z_k^B \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$. Consider $\| z_{k^*} \|_p$

$$= \sum_{j=1}^{n} |a_{k^*j} - \alpha_j| + \sum_{j=1}^{n} \sum_{l>j}^{n} | |a_{k^*j} - \alpha_j| - |a_{k^*l} - \alpha_l| |$$

$$= d \left(\sum_{j=1}^{n} \frac{|a_{k^*j} - \alpha_j|}{d} + \sum_{j=1}^{n} \sum_{l>j}^{n} | \frac{|a_{k^*j} - \alpha_j|}{d} - \frac{|a_{k^*l} - \alpha_l|}{d} |\right)$$

$$= d \| z_{k^*}^B \|_p$$

$$= d \min_{1\le k\le m} \| z_k^B \|_p \qquad \text{(by the assumption)}$$

$$= d \min_{1\le k\le m} \left(\sum_{k=1}^{n} |1 - \mu_{B_{kj}}(\alpha_j)| + \sum_{k=1}^{n} \sum_{l>j}^{n} | |1 - \mu_{B_{kj}}(\alpha_j)| - |1 - \mu_{B_{kl}}(\alpha_l)| |\right)$$

$$= d \min_{1\le k\le m} \left(\sum_{j=1}^{n} \frac{|a_{kj} - \alpha_j|}{d} + \sum_{j=1}^{n} \sum_{l>j}^{n} | \frac{|a_{kj} - \alpha_j|}{d} - \frac{|a_{kl} - \alpha_l|}{d} |\right)$$

$$= \min_{1\le k\le m} | z_k \|_p . \qquad \square$$

Moreover, it is clearly to obtain that $\| z_{k^*} \|_p < \| z_k \|_p$ for all $k \in \{1,2,...,m\} - \{k^*\}$ also.

Corollary 12 Let $\psi_A = [a_{kj}]_{m\times n}$ be a suitable value matrix over U. For each $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n) \in \mathbb{R}^n$ be the suitable values of a sample. For each $1 \le k \le m$ and $1 \le j \le n$, let B_{kj} be a fuzzy set on $[b_j, c_j]$. Let u_{k^*} be the optimal solution of the decision making problems. Then, there exists the particular membership function

$$\mu_{kj}(\alpha_j) = 1 - \frac{|a_{kj} - \alpha_j|}{d} \text{ for all } a_{kj}, \alpha_j \in [b_j, c_j],$$

where $d = \max_{1\le j\le n} |c_j - b_j|$ that satisfies u_{k^*} is the optimal solution in method III-A if and only if u_{k^*} is the optimal solution in method III-B.

Proof: The proof is complete from theorem 10 and theorem 11.

V. APPLICATIONS

In this section, the example about decision making problem is presented. It is shown that how method III-A and method III-B work. Moreover, it is shown also that these two methods have the same optimal solution via the particular membership function.

Example Assume that there is a set of 4 diseases $U = \{u_1, u_2, u_3, u_4\}$ with respect to the set of parameters $A = \{x_1$ is coughing, x_2 is fever, x_3 is weight loss, x_4 is nausea vomiting, x_5 is diarrhea$\}$. Suppose that each parameter has the range of getting disease symptoms; coughing 2-15 times per hours, fever 37.5-42 °C, weight loss 12-25 percent of normal weight, nausea vomiting 10-19 times per a day, and diarrhea 3-10 times per a day. Assume that the suitable values of U with respect to A are in the Table 2,

Table 2: The suitable values of diseases with respect to each parameter

Parameters / diseases	x_1	x_2	x_3	x_4	x_5
u_1	10	38	16	15	8
u_2	11	39	15	14	6
u_3	10	39	22	16	7
u_4	7	40	15	17	8

Assume that Mr.X has suffering from the illness. He goes to see doctor with the symptoms; coughing 9 times per hour, fever 39 °C, weight loss 15 percent of his weight, nausea vomiting 16 times a day, and diarrhea 7 times a day. *Which disease is he getting?*

From the assumption, we have that $\alpha = \{\alpha_1 = 9, \alpha_2 = 39, \alpha_3 = 15, \alpha_4 = 16, \alpha_5 = 7\}$.

From *the method III-A*, Sum norm and adjustable sum norm method based on suitable values.

$$[|M_{\psi_A} - \alpha|] = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 & 1 \\ 1 & 0 & 7 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 \end{pmatrix}$$

We obtain, $\| z_1 \|_s = 5, \| z_2 \|_s = 5, \| z_3 \|_s = 8, \| z_4 \|_s = 5$.

From the first step, we see that now we can't get the unique optimal solution. So, we have to go to the next step with computing adjustable sum norm.

We obtain, $\| z_1 \|_p = 5 + 0 = 5, \| z_2 \|_p = 5 + 12 = 17$, and $\| z_4 \|_s = 5 + 8 = 13$.

Thus, the disease u_1 is the optimal unique solution. Hence, one may predict that Mr.X is getting disease u_1.

From *the method III-B*, Sum norm and adjustable sum norm method based on fuzzy sets. Let a_{kj} be the suitable values of alternate k with respect to parameter j. And let d be the maximum width of the ranges from each parameter. Assume that the particular membership function is

$$\mu_{kj}(\alpha_j) = 1 - \frac{|a_{kj} - \alpha_j|}{d}$$

So, we get that

$$[|M^B_{\psi_A} - M^B_{\alpha}|] = \begin{pmatrix} 0.077 & 0.077 & 0.077 & 0.077 & 0.077 \\ 0.154 & 0 & 0 & 0.154 & 0.077 \\ 0.077 & 0 & 0.538 & 0 & 0 \\ 0.154 & 0.077 & 0 & 0.077 & 0.077 \end{pmatrix}$$

We obtain, $\| z_1^B \|_s = 0.385, \| z_2^B \|_s = 0.385, \| z_3^B \|_s = 0.615$, $\| z_4^B \|_s = 0.385$.

From the first step, we see that now we can't get the unique optimal solution. So, we have to go to the next step with computing adjustable sum norm.

We obtain, $\| z_1^B \|_p = 0.385 + 0.000 = 0.385$,

$\| z_2^B \|_p = 0.385 + 0.924 = 1.309$, and

$\| z_4^B \|_p = 0.385 + 0.616 = 1.001$.

Thus, the disease u_1 is the optimal unique solution. Hence, one may predict that Mr.X is getting disease u_1.

From calculation 2 methods, we get the same optimal solution under using the particular membership function.

VI. CONCLUSION

In this paper we have introduced an efficient procedures of solution, sum norm and adjustable sum norm method based on suitable values, and sum norm and adjustable sum norm method base on fuzzy sets in order to obtain an optimal solution of a fuzzy soft set based decision making problem. According to theorem 10 and 11, we can conclude that the problems for decision making by using sum norm and adjustable sum norm based one suitable values can be constructed a particular membership function to solve the problem that can be used in sum norm and adjustable sum norm based on fuzzy sets. It follows that these make both methods are equivalent to solve such a problem. However, sum norm and adjustable sum norm based on fuzzy sets is more efficiency than sum norm and adjustable sum norm based on suitable values because it can be used for any membership function with the sum norm and adjustable sum norm.

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