DYNAMICS OF PRICE STABILIZATION: CONTINUOUS- VERSUS DISCRETE-TIME COBWEB MODELS WITH BUFFER STOCK

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Abstract- This paper compared the dynamics of continuous-time nonlinear delay differential cobweb model with buffer stock to discrete-time nonlinear cobweb model with buffer stock, through simulation of real economic data of maize price demand and production in the Ashanti Region of Ghana. The nonlinear delay differential equation model, before the application of buffer stock scheme had price oscillated between two price points and could not converge, which affirms the condition of nonlinear model under naïve price expectation. It is established in the study that delay parameter is price stability dependent and also associated with price fluctuations. The nonlinear delay differential buffer stock model achieved price stability without varying the parameters values of supply function as opposed to discrete-time cobweb models.It was noted that, the more buffer stock delay and supply delay are well adjusted in connection with the price scheme run by buffer stock operators, the more stable the price becomes and the impact of buffer stock felt in the system. The results of the analysis provided an average stable price of maize as GH¢ 30.49 compared to the actual average price of GH¢30.27, which in turn determined the average stock during harvesting season. Nonlinear discrete-time cobweb model on the other hand, could not meet the naïve price expectation condition until parameter values were reviewed. This made the predicted values of average price and stock unrealistic. It was found that, discrete-time buffer stock model performed well under linear models than in the nonlinear models, which is a limiting case of the delay buffer stock model used in this study. It is therefore, presumed from the study that researchers would rather use continuous-time nonlinear delay models as they reflect realities prevailing in most market economic problems.

Keywords- buffer stock, price stabilization, delay-differential equation, cobweb models

I. INTRODUCTION

The price maize in recent times tends to fluctuate more than prices of manufactured products and services. This is largely due to the distortion in production and distribution of agricultural products and also with the fact that demand and supply are price inelastic (Riley, 2012; Selvi, 2014). It is believed that, price volatility and scarcity of maize is a great threat in the distribution of this staple food since in Ghana, as in other developing countries, maize usually accounts for quite significant share of poor households' budgets (Wright, 2001; Sutopo et al, 2012). It is no wonder the disparities in terms of supply during the harvest and planting seasons create inconveniences to stakeholders such as producers, consumers and government (Sutopo et al, 2012).

The poor have limited possibilities to insure against adverse price shocks and therefore an implementation of food price stabilization policies is the only means by which governments in the developing countries can address the concerns regarding price instability problem (Wright, 2001).

1.1 Buffer Stock Scheme

Buffer stock scheme is one of the mechanisms through which price fluctuations could be smoothened out. This scheme seeks to stabilize the market price of agricultural products by buying up supplies of the product when harvests are plentiful and selling stocks of the product onto the market when supplies are low (Riley, 2012).

Myers et al. (1989) paint a relatively favourable picture of the buffer stock scheme such that in its absence, demand shocks are the dominant source of variability, but when stockholding implemented the effects of these shocks are blunted. It is considered that stockholding is commodity price stabilizer as it forms an intriguing characteristic of price dynamics as found and explained by Mitra and Boussard (2011). It is also proven that nonlinear cobweb model with endogenous factors such as storage, risk averse agents and adaptive price expectations is very efficient in curbing price volatility (Mitra and Boussard 2011). Sutopo et al.(2009) also incorporated nonlinear buffer stock model in a composite model comprising optimization and econometrics techniques to achieve price stability using parameters such as quantity, time and price.

Anokye and Oduro (2013) also developed linear cobweb model to study the phenomenon of tomato price fluctuations and the results showed unstable price oscillations around the equilibrium point. However, after incorporating buffer stock model, stable price was achieved in the short run but suffered instability in the long run, until average supply was reviewed.

Soltes et al. (2012) also applied continuous-times buffer stock model in which price tend to an equilibrium point, not only monotonically but also oscillates around the equilibrium point, and they attributed the cause of the oscillations to the order of the differential equations.

Delay differential buffer model was used by (Anokye and Oduro, 2014) to study the impact of delay on price and the results showed that the efficiency of a buffer system is dependent on delay variation that is suitably enough and jointly connected to buffer stock and supply systems.

All the buffer stock models reviewed so far by the study have failed to consider the effects of delay (time lag) on price and supply change in response to market dynamics and also on buffer stock operation in stabilizing prices of food commodities, except that of (Anokye and Oduro, 2015). Therefore, the study is intended to compare the effect of continuous-time delay differential model with buffer and discrete-time model with buffer on price of maize in Ghana, so as to be able to establish the most robust model to be applied to control prices and yet expect the model to reflect the realities prevailing at the market.

Thus the models would be expected to determine the average price, average stock and supply needed to create stable market and also assist in investigating the causes of price fluctuations in the context of cobweb model with buffer stock.

II. MATERIALS AND METHODOLOGY

In this study record of price and production of maize from seventeen (17) major market centres in the Ashanti Region, Ghana was used. This secondary data covered the years 1994 to 2013, and was collected by the Ministry of Food and Agriculture, Statistical Directorate Kumasi-Ghana. Refer to table A1, in the appendix.

The table A1, contains the quarterly price and production of maize data at the right and left sections of the table respectively. They are average price and production points across the seventeen (17) market centres.

Also in the appendix is table A2, which displays the statistical characteristics of the data contained in this table A1; Table A2 shows descriptive statistics of the data in table A1, which includes range, mean, skewness over 1 (moderate), kurtosis greater than 0 but within the expected value of 3 and their respective standard errors.

2.1 Continuous-time non-linear Model with Buffer Stock

The study consider a simple nonlinear delay differential equation (up to quadratic term) for the supply function of price as,:

$$S(p(t)) = b + \beta\, p(t-\tau) - \delta\, p^2(t-\tau) \qquad (1)$$

and linear demand function of price as

$$D(p(t)) = a - \alpha\, p(t) \qquad (2)$$

where a, b, α, β and τ are positive constants. The delay in the supply function is the time length of τ, within price decisions are taken before the supply is done.

Assumption

The market price is also assumed to be determined by only the available supply in a single market and the rate of change of the price is proportional to the difference between supply and demand functions (1)-(2) (Mackey, 1989; Fulford, 1997), to provide the following function;

$$p'(t) = [(a-b) - \alpha\, p(t) - \beta\, p(t-\tau) + \delta\, p^2(t-\tau)] \qquad (3)$$

where, τ>0, on [0, b], b>0

with history function defined by p(t) = s(t), on [-τ , 0]. This is a simple nonlinear delay differential equation which mimics discrete time nonlinear difference equation modelled by (Jensen and Urban, 1984).

Buffer stock model is incorporated into equation (3), to obtain equation (4) given by:

$$p'(t) = \big[D(p(t)) - S(p(t))\big] - \int_{\tau}^{t} \big[S(p(u)) - D(p(u))\big]du \qquad (4)$$

where the second term is the integral of past differences which constitutes the accumulated stock (say G) in the buffer.

This price adjustment model (equation 4) mimics the undelayed integro-differential model used by Soltes et al. (2012), and can be transformed into differentiation by simplifying it into the form given by:

$$p''(t) = \big[D'(p(t)) - S'(p(t))\big] - \big[S(p(t)) - D(p(t))\big] \qquad (5)$$

The following equation is obtained when the equation (3) fixed into the equation (5);

$$p''(t) = p'(t) - \begin{bmatrix} (b-a) + \alpha\, p(t) + \beta\, p(t-\tau) \\ -\delta\, p^2(t-\tau) \end{bmatrix} \qquad (6)$$

where $p'(t)$, is equation(3), and equation (5) defined by the same history function p(t) = s(t), on [-τ , 0], where the solution sequence holds on the interval [t-τ, t[t, t+τ], [t+τ, t+2τ], etc., from time points t = 0, τ, 2τ,..., using Method of Steps (Roussel, 2005).

If G>0, then it adjusts price downwards because stocks are released to the market. If G<0, then stocks are bought from the market to adjust price upward G = 0 denotes no interference from the buffer stock scheme (Athanasiou et al., 2008; Soltes et al., 2012; Anokye and Oduro, 2013).

In general, it is very difficult to apply analytical solution (3) or (6) using method of steps and therefore numerical solution is recommended using Matlab solver dde23 that applies the principles of Runge Kutta triple BS(2,3) similar to the approach of method steps using.

2.2 Discrete time non-linear model with Buffer Stock

In considering a linear difference equation or demand function of price of the form given as:

$$D_t = \alpha - \beta\, p_t \qquad (7)$$

with a simple non-linear difference equation or supply function of price also given as;

$$S_t = \delta + \gamma p_{t-1} - \rho p_{t-1}^2 \qquad (8)$$

then at price equilibrium, (7) = (8), and the following nonlinear difference equation that constitutes nonlinear cobweb model, similar to that of (Jensen and Urban, 1984), is obtained:

$$p_t = C - Bp_{t-1} + Ap_{t-1}^2 \qquad (9)$$

where $C = (\alpha - \delta)/\beta$, $B = \gamma/\beta$ and $A = \rho/\beta$

After applying lagging operator on (9), the solution of equation (9) will have two fixed points as follows;

$$p_t = \frac{(1+B) \pm \sqrt{(1+B)^2 - 4AC}}{2A}$$

provided $(1+B) - 4AC \geq 0$

This solution is similar to quadratic equation which has two fixed points and therefore,

If $\frac{(1+B)^2}{4A^2} - \frac{C}{A} > 0$,

then two equilibrium points exist where the smaller one is a sink, and the bigger one a source.

If $\frac{(1+B)^2}{4A^2} - \frac{C}{A} = 0$,

and that two repeated eqilibria obtained as:

$$p_{1,2} = \frac{(1+B)}{2A}$$

and at this point bifurcation occurs, and so any point in it's neighbourhood will be attracted from below and be repelling from above.

Buffer stock model is incorporated into (9) in the form of supply fixed at the average supply. It is assumed that stabilization is guaranteed towards the average price (Anokye and Oduro 2013; Athanasiou et al., 2008).

Given that the average quantity of maize supplied at last time t is expressed by:

$$S_t^A = \frac{\sum_{j=1}^{k} S_{t-j}}{k} + G_{t-1} \qquad (10)$$

where G_{t-1} is a control variable which constitutes the accumulated stock in buffer.

At equilibrium

$$D_t = S_t + G_{t-1} \qquad (11)$$

where

$$G_{t-1} = S_t^A - g(p_{t-1}^e)$$

with $g(p_{t-1}^e)$ as the estimated supply at period t.

When equations (7) and (8) are substituted in the equation (11), the following nonlinear difference is obtained:

$$p_t = Ap_{t-1}^2 - Bp_{t-1} + C - (S_t^A - g(p_{t-1}^e)) \qquad (12)$$

where $C = \frac{(\alpha - \delta)}{\beta}$, $B = \frac{\gamma}{\beta}$, $A = \frac{\rho}{\beta}$, and $D = \frac{1}{\beta}$,

if $G_{t-1} > 0$, it adjusts price downwards because stocks are released to the market at any period t,

if $G_{t-1} < 0$, it adjusts price upwards since stocks are bought to the buffer at period t.

III. FINDINGS

The results of the analysis in twofold are discussed with respect to discrete-time model and continuous-time model as follows:

3.1 Discrete-Time Model

The models (equations 7 and 8) are fixed with estimated parameter values obtained through regression using price and supply data of maize from appendix A.1 and the following price function obtained;

$$D_t = -96.16\,p_t \quad \text{where } \alpha = 0 \qquad (13)$$

$$S_t = 354.28\,p_{t-1} - 2.78\,p_{t-1}^2 \quad \text{where } \delta = 0 \qquad (14)$$

At the market equilibrium, equations (13 and 14) provide the following nonlinear difference equation that constitutes nonlinear cobweb model similar to equation (9) with C = 0, B = -3.68 and A = 0.03;

$$p_t = -3.68\,p_{t-1} + 0.03\,p_{t-1}^2 \qquad (15)$$

By applying lagging operator and solving (15) two price fixed points are obtained;

$$p_{1,2} = \frac{(4.684) \pm \sqrt{4.684^2}}{2(0.03)}$$

where $p_1 = 161.52$ is a source or unstable since $\left|p_1'(161.52)\right| > 1$ and

$p_2 = 0$ is also source or unstable since $\left|p_2'(0)\right| > 1$

The bifurcation occurs at $p_k = 80.75$, which is very attractive from below and repelling from above. Thus price will oscillate between the two price points.

Numerical Approach:

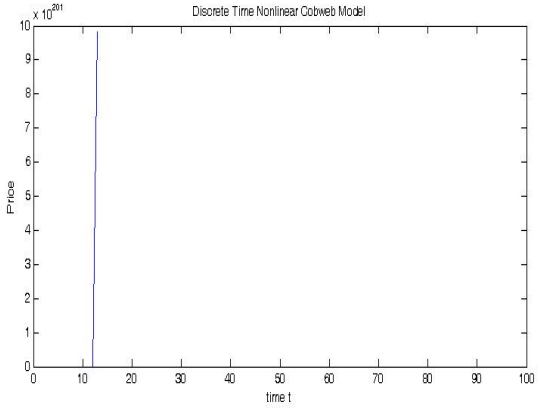


Figure : 1: Oscillation of Price around Equilibrium point

From figure 1, it is clear that price stability of maize can only be achieved in short term at $p_2 = 0$ and since producers are sensitive to price, they would be attracted towards any other price or unstable equilibrium price $p_1 = 161.52$ as demonstrated in the analytical solution.

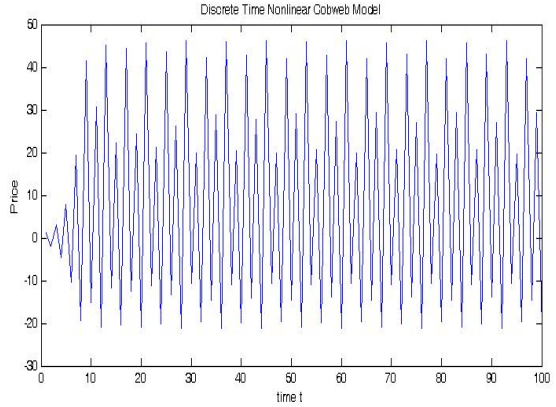


Figure : 2: Oscillation of Price around Equilibrium point with $\left|\frac{\gamma}{\beta}\right| > 1 < B$

Figure 2, shows that when marginal supply is reduced to 156.12 from 354.28 (refer to equation 14), and marginal demand maintained at 96.16 so that $\left|\frac{\gamma}{\beta}\right| > 1 < B$, (also referred to as ratio of marginal supply over marginal demand) the oscillatory condition of nonlinear cobweb model under naive price expectation is confirmed.

Thus the steady state is unstable and prices diverge from equilibrium and converge to a (stable) 2-cycle (Ezekiel,1938; Hommes,2013; Irma et al.,1999).

3.1.2 Cobweb Model with Buffer Stock

Using equation (12) with supply fixed at the average $S_t^A = 16008.72$,and C = 0, B = -3.68, D = 0.0104 and A = 0.03 then;

$$p_t = 0.03\,p_{t-1}^2 - 3.68\,p_{t-1} - 0.0104(S_t^A - g(p_{t-1}^e)) \quad (16)$$

where the estimated supply $g(p_{t-1}^e)$ at t is the negation of supply function at t with $g(p_0^e) = 0$ at t = 1.

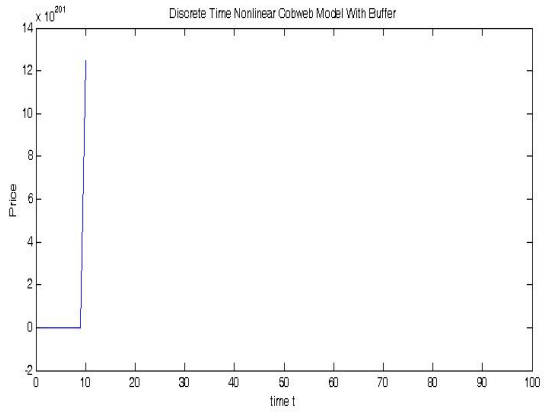


Figure : 3: Price Oscillation with Buffer Stock at Average Supply

From figure 3, the price behaviour of maize is similar to that of figure 1, producers are attracted towards the unstable equilibrium price $p_1 = 161.52$ or any other price instead of the zero equilibrium price ($p_2 = 0$). The buffer stock seems to have no effects on the price.

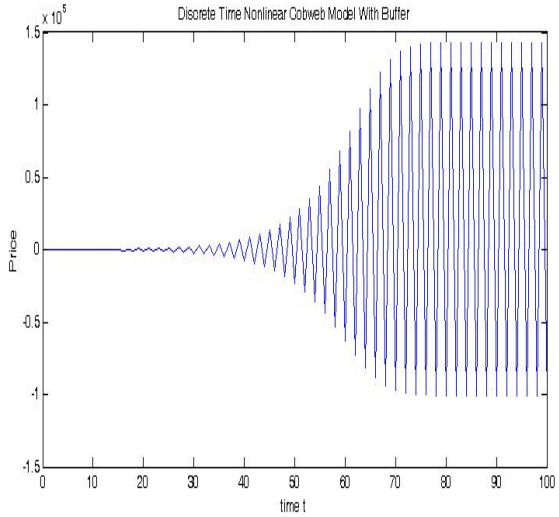


Figure 4: Buffer Stock at Average Supply with $\left|\frac{\gamma}{\beta}\right| > 1 < B$ **and** $\rho < 1$

It is clear from figure above, that if the coefficient of quadratic term and the marginal supply are both reduced (i.e $\gamma = 106.92$ from 354.28 and $\rho = 0.00028$ from 2.78) refer to supply function of price (equation 14), price stability is achieved but only for a short term.

3.2 Continuous-Time Model

After fixing the same estimated parameter values into the equations (1 and 2), the following equations are obtained;

$$S(p(t)) = 354.28\,p(t-\tau) - 2.78\,p^2(t-\tau)$$

where b = 0 (17)

$$D(p(t)) = -96.16\,p(t) \quad \text{where a = 0} \quad (18)$$

The rate of change of price (being difference between equations (17)-(18)) is given by the following equation, in the form of equation (3) with τ =1 and t>0;

$$\tfrac{1}{96.16}\,p'(t) = \left[-p(t) - 3.68\,p(t-1) + 0.03\,p^2(t-1)\right] \quad (19)$$

with history function p(t)=1.23 for $t \leq = 0$, and sequence of solutions on interval say [0, 10]. Using the method of steps which involves solving the equation on one interval at a time, then for [0, 1];

$$\int_0^t p'(s)ds = p(s)\Big|_o^t = p(t) - p(0)$$

$$p(t) = p(0) + \int_0^t p'(s)ds$$

If $p'(t) = p(t-1)$, it implies $p'(s) = p(s-1)$, and so

$$p(t) = p(0) + \int_0^t p(s-1)ds$$

Then on the interval $0 \le s \le 1$, implies $-1 \le s-1 \le 0$, so that history function $p(s-1) = 1.23$, on this interval;

$$p(t) = 1.23 + \int_0^t [-1.23 - 3.68(1.23) + 0.03(1.23)^2]ds$$
$$p(t) = 1.23 + (-5.72)s\Big|_0^t$$
$$p(t) = 1.23 + (-5.72)t$$
(20)

To find the next solution of equation (19), on the interval [1, 2], the solution from [0, 1] is used and so using same process;

$$p(t) = p(1) + \int_0^t p'(s)ds$$

The solution on [0, 1] from equation (20) is given by

$$p(1) = 1.23 + (-5.72)(1) = -4.49$$

Then on the interval $1 \le s \le 2$, also implies $0 \le s-1 \le 1$, so that previous solution would be $p(s-1) = 1.23 + (-5.72)(s-1)$, so on this interval;

$$p(t) = -4.49 + \int_0^t p(s-1)ds$$
$$p(t) = -4.49 + \int_0^t [1.23 + (-5.72(s-1))]ds$$
$$p(t) = -4.49 + [6.95s - \tfrac{5.72s^2}{2}]_1^t$$
$$p(t) = -\tfrac{17.16}{2} + 6.95t - \tfrac{5.72t^2}{2}$$

It is noted that each new step, adds a new piece and the solutions tend to get more complicated over each successive interval. Therefore on the interval [0,10], one obtains a piecewise function with ten distinct pieces. The process is continued until unique solution is obtained and this solution must also satisfy the initial conditions as well as the initial value problem (equation 19).

Continuity Analysis

$$\lim_{t \to 1^+} p(t) = (1.23 + (-5.72)t) = -4.49$$
$$\lim_{t \to 1^-} p(t) = (-\tfrac{17.16}{2} + 6.95t - \tfrac{5.72t^2}{2}) = -4.49$$

The piece-wise functions are continuous in the given interval since the left hand limit is equal to the right hand limit and therefore one resort to numerical approach on assumption that continuity holds in all intervals.

Numerical Solution:

Using MatLab solver dde23 (code attached at appendix 2), in solving equation (19), with solution interval now increased to [0; 100], the following solution obtained;

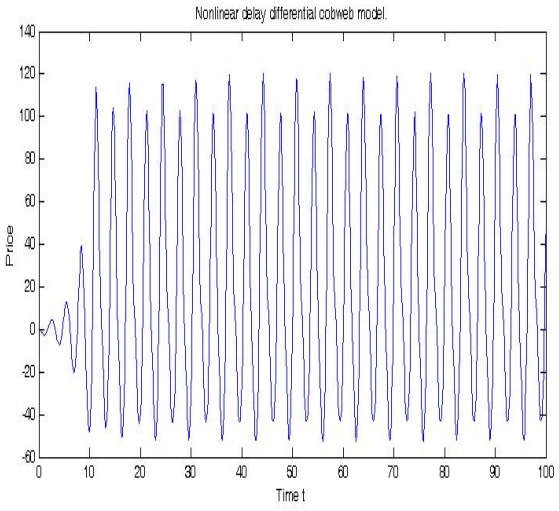


Figure 5: Oscillations of Price around 2 Equilibrium Points for $\tau = 1$

From the Figure 5, the solution of the system oscillates between two (2) equilibrium (price) points, which conforms to the condition of nonlinear models under naive price expectation (Ezekiel,1938; Hommes,2013; Irma et al., 1999).

3.2.1 Continuous-Time Model with Buffer Stock

From equation (6), the buffer stock equation is obtained as follows with b = a = 0:

$$\tfrac{1}{96.16} p''(t) = [(-p(t) - 3.68(t-\tau) + 0.03p^2(t-\tau))']$$
$$-[p(t) + 3.68p(t-\tau) - 0.03p^2(t-\tau)]$$
(21)

If delay (τ) is fixed at 0.45 for both the buffer stock and supply system, then the following solution is obtained:

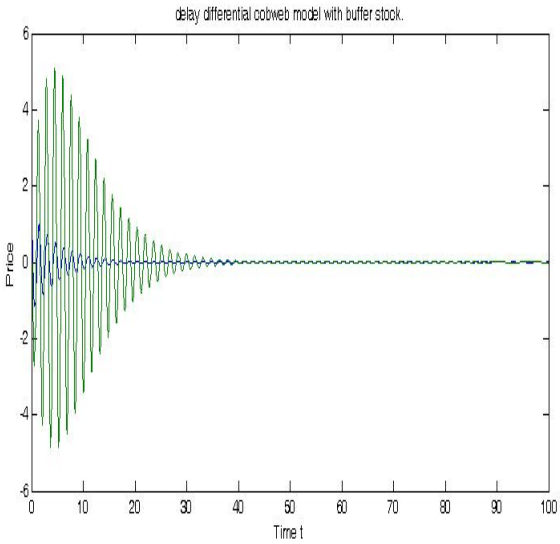


Figure 6: Oscillations of Price about equilibrium with Buffer Stock, $\tau = 0.45$

From figure 6, above, the buffer stock and the supply are of different wave length which signifies no effects on price from the buffer operation. The two systems should synchronize in order to have effect on the price.

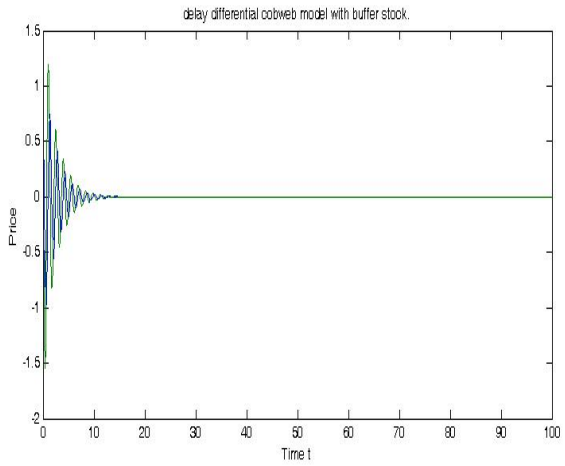


Figure 7: Oscillations of Price about Equilibrium with Buffer Stock, $\tau = 0.21$

Figure 7, gives best delay times of $\tau_b = 0.21$ for buffer stock and $\tau_s = 0.40$ for supply system to provide average equilibrium price of GHC 30.49 (from simulated mean price of GHC 0.317 x 96.16) per mini bag; very close to the actual mean price of GHC 30.27. This GHC 30.49 in turn provided average stock of 5286 metric tons from demand of 2931.6 metric tons and supply of 8217.6 metric tons.

DISCUSSION AND CONCLUSIONS

It is found that the results of discrete-time models support the assertion by Jensen and Urban (1984) that nonlinear models achieve stability only within certain parameter boundary and beyond the boundary exhibit spectrum of behaviour including chaos.

However, changing of parameter values in supply function, affects the behaviour of the model as well as the price, supply and quantity of stock values predicted from the model; they would not reflect the true market circumstances.

The nonlinear delay differential equation model, before the application of buffer stock scheme had price oscillated between two price points and could not converge, which affirms the condition of nonlinear model under naïve price expectation.

It is established in the study that delay parameter is price stability dependent and also associated with price fluctuations. It is also found that nonlinear delay differential buffer stock models achieve price stability without varying the parameters values of supply function as opposed to discrete-time cobweb models.

Nonlinear delay differential buffer models also determined average price and stock together with average demand and supply, by taking decision at stock level before supply which is limiting case in discrete-time buffer models.

It was noted that, the more buffer stock delay and supply delay were well adjusted in connection with the price scheme run by buffer stock operators, the more stable the price becomes and the impact of buffer stock felt in the system. The results of the analysis provided an average stable price of maize as GH¢ 30.49 compared to the actual average price of GH¢30.27, which in turn determined the average stock during harvesting season.

The findings of the study are interpreted in line with its limitations, of which some could be addressed in further studies to evaluate the impact of buffer stock on price when private storage is allowed to compete with the government. And also when more than one market is modelled to see how price stabilization could be guaranteed in-between two or more markets. Since it is assumed that when farmers reduce their production of one produce because they expect a lower price, they are likely to increase their production in the same or another produce in another market, where price is favourable.

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APPENDICES

Appendix 1

Table A1: Price and Production of Maize in Ashanti Region

	Prices of Maize (Bag of 100 kg) in GH¢				Production of Maize (Metric Tons)				
Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
1994	1.23	1.78	1.49	1.67	10198.13	14830.01	12403.34	13878.19	51309.67
1995	2.69	3.53	1.64	2.22	15947.12	20917.51	9730.17	13171.87	59766.67
1996	2.80	3.41	4.61	4.45	10152.87	12357.34	16710.75	16125.71	55346.67
1997	6.51	8.73	6.74	-	17268.13	23180.01	17886.86	-	58335.00
1998	5.06	5.62	4.43	4.26	19354.45	21514.8	16930.53	16287.52	74087.30
1999	4.64	4.94	5.76	4.60	15016.85	15978.78	18646.42	14875.28	64517.33
2000	9.32	11.05	9.90	10.39	14663.71	17381.97	15579.11	16342.87	63967.66
2001	13.87	18.76	12.98	13.04	13403.83	18127.28	12539.71	12595.85	56666.67
2002	14.83	15.38	10.97	11.36	25347.56	26300.98	18760.61	19417.51	89826.66
2003	14.03	17.06	16.32	14.49	14649.34	17818.76	17039.9	15132	64640.00
2004	17.80	21.67	23.46	23.06	12629.82	15372.77	16646.07	16362.01	61010.67
2005	29.67	45.02	34.25	28.56	11638.64	17660.19	13436.17	11203.67	53938.67
2006	25.91	27.39	20.98	18.87	15226.02	16097.1	12328.32	11090.56	54742.00
2007	25.85	32.34	26.41	26.06	13190.26	16500.28	13474.43	13296.04	56461.01
2008	32.34	56.66	56.72	49.39	10103.38	17700.44	17717.1	15428.41	60949.33
2009	60.79	71.77	55.13	52.21	15780.74	18631.95	14311.43	13552.55	62276.67
2010	53.29	55.51	52.76	46.82	21597.84	22497.63	21385.72	18976.81	84458.00
2011	55.10	75.41	81.34	89.95	10572.89	14470.08	15607.96	17260.74	57911.67
2012	110.69	124.44	90.97	76.00	18169.81	20426.27	14932.32	12475.61	66004.01
2013	77.09	76.24	74.86	81.67	17036.49	16847.17	16542.22	18047.12	68473.00

Table A2: Descriptive Statistics of Data in Table A1

	N	Range	Mean		Std. Deviation	Skewness		Kurtosis	
	Statistic	Statistic	Statistic	Std. Error	Statistic	Statistic	Std. Error	Statistic	Std. Error
Price	79	123.21	30.27	3.31709	29.48293	1.160	.271	.584	.535
Production	79	16570.81	16008.72	383.99735	3413.04306	.556	.271	.604	.535
Valid N (listwise)	79								

Appendix 2

PROGRAM CODE

```
function ddebuffer2
%DDEPRICE Solution for DDE23.
% This code for delay differential equations with buffer stock % ddeprice are solved on [0, 100] with history y_1(t) = 1.23 and y'_1(0) = 0 ,for t <= 0.
% the buffer used the same history function
% The lags are specified as a vector [a,b] (they are varied), the delay differential
% equations are coded in the subfunction DDEPRICE, and the history is
% evaluated by the function histprice. Because the history is constant it
% could be supplied as a vector:
% sol = dde23(@ddeprice,[a,b],[1.23;0],[0, 100]);
```

```
sol = dde23(@ddebuffer,[0.40,0.21],@histbuffer,[0, 100]);
figure;
plot(sol.x,sol.y)
title('delay diff cobweb model with buffer stock.');
xlabel('time t');
ylabel('solution y');
x = linspace(0,20,100);
y = deval(sol,x,1);
% ------------------------------------------------------------
function s = histbuffer(t)
% Constant history function for ddeprice.
s = [1.23,1.23];
% ------------------------------------------------------------
function dydt = ddebuffer(t,y,Z)
% Delay Differential equations function for DDEPRICE.
ylag1 = Z(:,1);
ylag2 = Z(:,2);
dydt = [ (-1*y(1)-3.68*ylag1(1)+0.029*ylag1(1)^2);
((-1*y(1)-3.68*ylag1(1)+0.029*ylag1(1)^2)-
(1*y(2)+3.68*ylag2(2)-0.029*ylag2(2)^2))];
```

★★★